

1967

The crystal structure determinations of $OP(OCH_2)_3CCH_3$ and a hydrolysis product of $PO_3C_6H_9$

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NIMROD, Dale Meyers, 1940-
THE CRYSTAL STRUCTURE DETERMINATIONS OF
OP(OCH₂)₃CCH₃ AND A HYDROLYSIS PRODUCT
OF PO₃C₆H₉.

Iowa State University, Ph.D., 1967
Chemistry, physical

University Microfilms, Inc., Ann Arbor, Michigan

THE CRYSTAL STRUCTURE DETERMINATIONS OF $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$
AND A HYDROLYSIS PRODUCT OF $\text{PO}_3\text{C}_6\text{H}_9$

by

Dale Meyers Nimrod

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Physical Chemistry

Approved:

Signature was redacted for privacy.

In Charge of Major Work

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Head of Major Department

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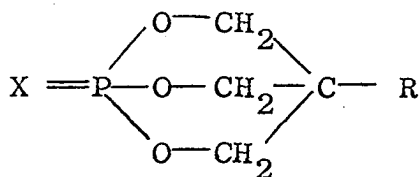
1967

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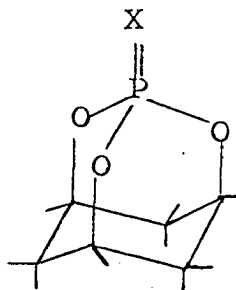
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INTRODUCTION

The chemistry of bicyclic alkoxy phosphorus compounds of the types



I



II

where X = nothing, O or S, and R = CH₃, C₂H₅ or n-C₅H₁₁, has been the subject of considerable study since the compounds were first reported. The phosphite forms have received the most attention due to their strong ligand properties. Interestingly high P = O stretching frequencies have been observed in the infrared spectra of the phosphate forms. The mechanisms for opening the caged structures with alkyl halides, water and halogens have been compared and contrasted. Nuclear magnetic resonance and dipole moment experiments as well as infrared studies have been used as tools for postulating structural features of the compounds and their derivatives.

The crystal structure determinations of two of these compounds were undertaken by means of x-ray diffraction to

confirm the postulated configurations and to measure quantitatively their various structural features. The compounds examined in this study were the 4-methyl-1-oxide derivative of I, 1-oxo-4-methyl-2,6,7-trioxa-1-phosphabicyclo[2.2.2]octane, hereafter referred to as phos-oxide, and one of the two isomers of the acid-catalyzed hydrolysis of the phosphite form of II, 3- α -oxo-3- β -hydrido-7- β -hydroxy-2,4-dioxa-3-phosphabicyclo[3.3.1]nonane, hereafter referred to as isomer A.

In addition the space group determination and its structural implications of the crystal structure of the related bicyclic phosphorus compound, $OP(N(CH_3)CH_2)_3CCH_3$ (1-oxo-2,6,7-trimethyl-4-methyl-2,6,7-triaza-1-phosphabicyclo[2.2.2]octane) are reported in Appendix A.

In the course of automating a four-circle x-ray diffractometer by means of a computer system an algorithm for maximizing and centering a general reflection was developed. The mathematical expressions necessary for its implementation were derived and are presented in Appendix B.

THE STRUCTURE OF $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$

Introduction

Bicyclic alkoxy phosphites of the types $\text{P}(\text{OCH}_2)_3\text{CR}$, where $\text{R}=\text{CH}_3$, C_2H_5 and $n\text{-C}_5\text{H}_{11}$, and $\text{P}(\text{OCH})_3(\text{CH}_2)_3$ have been shown to function as excellent electron donors in a variety of conditions and have attracted a good deal of attention since their discoveries (1,2). The unoxidized parent of the subject of this structure determination, $\text{P}(\text{OCH}_2)_3\text{CCH}_3$, has been a particularly strong ligand. In contrast with comparable open chain trialkyl phosphites which form complexes with only the heavy transition metals such as $\text{Pt}(\text{II})$ and a few post transition metals such as $\text{Hg}(\text{II})$, $\text{P}(\text{OCH}_2)_3\text{CCH}_3$ forms complexes with first row transition metal ions (3) and with transition metal carbonyls (4,5) as well as stable adducts with boron containing Lewis acids (6).

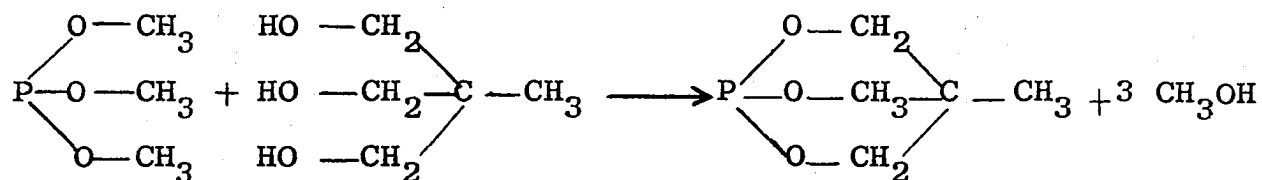
The H^1 and P^{31} n.m.r. spectra of both the phosphite and the phosphate were investigated by Verkade and King (7) and gave firm support to the postulated bicyclic configuration. The simplicity of the proton spectra, consisting of a methyl singlet and a methylene doublet, could be accounted for only with the presence of a threefold axis through the molecule. The P^{31} resonance and coupling also supported the configuration.

The greater donor ability of the bicyclic phosphite is undoubtedly due to the reduction of steric hinderance between the ligands in a complexed system as well as between the ligand itself and the metal to which it is coordinated when the alkoxy groups are constrained in the manner in this cyclic ring system.

In spite of the fact that the molecular configuration had been deduced, the x-ray structure analysis of phos-oxide was undertaken for several reasons. First of all, no other bicyclic phosphate structure determinations have been reported. The structural parameters of the phosphate are expected to closely approximate the structural features of the coordinated phosphite in transition metal complexes and adducts which have been and are under study. Quantitative estimates of bond angles on the basis of n.m.r. spectra have been made and should be tested. The structural parameters of phosphonium cations of the type $[\text{RP}(\text{OCH}_2)_3\text{CCH}_3]^+$ (8) and of phosphonium intermediates in the Michaelis-Arbuzov reactions of $\text{P}(\text{OCH}_2)_3\text{CCH}_3$ (9) should be closely approximated by the structure of phos-oxide. In addition the structural parameters were sought to see if the lack of extraction properties of the related bicyclic phosphate, $\text{OP}(\text{OCH}_2)_3\text{CC}_5\text{H}_{11}$, could be understood (10). Related to the last point is the question concerning the reason for the very high P=O infrared stretching frequencies observed in the bicyclic phosphates and their implications in terms of bonding.

Experimental

The preparation of the unoxidized form of phos-oxide was accomplished by refluxing trimethyl phosphite with the proper alcohol (2-hydroxymethyl-2-methyl-1,3-propanediol) (8).



Oxidation to the phosphate takes place with the addition of hydrogen peroxide to a solution of the phosphite in absolute ethanol (1).

Crystals of phos-oxide suitable for x-ray analysis were obtained by slow sublimation at 100°C inside a water cooled sublimation apparatus. The long, needle-like, soft crystals which formed on the wall of the apparatus were removed and severed with a razor blade into approximate cubes. Torsional distortion invoked by the stress of the cutting process was a problem, but the degree of distortion could be observed by examining Weissenberg photographs for drawn-out spots along the axis of rotation (the crystal needle axis). A good single crystal which apparently was not stressed was obtained by this procedure and was the crystal used for the intensity measurements. Later attempts, however, to get suitable fresh crystals by this method for the purpose of obtaining

accurate lattice constants and for remeasuring a portion of the intensity data were not successful.

Other attempts to grow suitable crystals included sublimation inside an evacuated seven millimeter glass tube. Heat was supplied in one case with a 500 watt heat bulb and in another case with a coil of resistance wire wrapped around the tube so as to make a smooth temperature gradient. Considerable difficulty, however, was encountered in attempting to remove the crystals without damaging them since they tended to adhere firmly to the glass wall. Attempts to grow suitable crystals from methanol, ethanol, butanol, and dioxane solutions were also unsuccessful.

The crystals used for the space group determination as well as for the intensity gathering process were glued onto the end of a glass fiber with the needle axis being coincident with the mounting axis. The long axis of the unit cell, c , is coincident with the external needle axis.

Both Weissenberg and precession photographs were taken with $\text{CuK}\alpha$ radiation. The observed diffraction symmetry was $m m m$ (orthorhombic). Systematic conditions for non-extinction were observed to be as follows:

hkl : no conditions

$h0l$: $l=2n$

$00l$: $l=2n$ (redundant)

$h00$: $h=2n$.

The first three conditions are compatible with space groups Pmcm [Pmma as defined in the International Tables (11)], P2cm (Pma2) and Pmc2₁. The third condition is not required by the space group; the implication of its presence will be discussed later.

Measurements for the determination of the lattice parameters of phos-oxide were taken from several sources. In addition to measurement made on the Weissenberg and precession photographs (CuK α radiation) a General Electric single crystal orienter x-ray unit was used with both MoK α and CrK α radiation. Three more measurements were taken using a Hilger and Watts four-circle x-ray diffractometer using MoK α radiation. The full circle capabilities allowed the measurement of the same reflection in several equivalent orientations. The data were refined and the lattice parameters and their estimated standard deviations were calculated by means of a least-squares lattice constant refinement program of Williams (12). The parameters and their standard deviations taken from the variance-covariance matrix are as follows:

$$a = 6.736 \pm 0.003 \text{ \AA}$$

$$b = 5.960 \pm 0.017 \text{ \AA}$$

$$c = 17.768 \pm 0.004 \text{ \AA}$$

$$\text{volume} = 713.3 \text{ \AA}^3.$$

Four molecules (molecular weight = 164.1) per unit cell give a calculated density of 1.53 gms/cm^3 . The observed density, obtained by means of flotation observations of single crystals in mixtures of benzene and carbon tetrachloride, of 1.51 gms/cm^3 agrees favorably.

Two experiments were carried out in an effort to elucidate the presence or absence of a center of symmetry in the space group. Maurice (13) described a test in 1930 whereby a single crystal is immersed in a bath of liquid nitrogen. Non-centrosymmetric crystals may become electrostatically charged, and upon removal, exhibit whisker-like crystals of ice following the lines of electric force in the same manner in which iron filings follow the magnetic lines of force around a magnet. On the other hand, since no electrostatic charge can be built up in centrosymmetric crystals, the ice forms an even coat on the crystal.

The largest single crystal of phos-oxide that could be found was glued to the end of a fine thread and suspended in a bath of liquid nitrogen. A microscope, previously focused on the crystal, was used to observe the icing when the liquid nitrogen was removed. It appeared as if the icing formed an even coating on the crystal. Control experiments were run with α -quartz, a well known acentric crystal, upon which ice formed in a pattern definitely following the lines of the electric field. Positive tests (a smooth coating) were

obtained for two known centric crystals. It was recognized, however, that although a positive test for pyroelectricity is a valid indication of a non-centrosymmetric crystal, the lack of a display of pyroelectricity cannot be taken as a true indication for the presence of a center of symmetry in the crystal. No space group implications, therefore, could be made on the basis of this test.

A second experimental test for pyroelectricity as described by Wheatley (14) was attempted in which a powdered sample is placed in an anodized aluminum spoon to which liquid nitrogen is added. As the nitrogen evaporates the crystals form a ball which either collapses or remains together depending upon whether the crystals are centrosymmetric or non-centrosymmetric respectively. The phos-oxide crystals were too soft and waxy to form a suitable powder, however, and no conclusions could be made from this approach, either.

Examination of the symmetry elements of the three possible space groups coupled with a knowledge of the number of molecules per unit cell, a well-grounded proposal for the molecular configuration, and the non-required systematic extinctions in the $h00$ data, led to the assignment of space group $Pmc2_1$. That the extra systematic extinction does not correspond to a twofold screw axis in the x direction can be seen from group theory. Since the product of any two elements of the group must be a member

of the group, and since a c glide in the b direction is known to exist, the product of the operations of the screw axis and the glide plane must be in the group. A c glide in the b direction takes a general point, (x,y,z) , to $(x,-y, 1/2 + z)$. A twofold screw axis in the a direction takes the result to $(x + 1/2, y, 1/2 - z)$ which is the point reached by operating on (x,y,z) with an a glide in the c direction when the glide plane is located at $z = 1/4$. Now an a glide in the c direction would result in systematic extinctions in the $hk0$ data for all reflections for which h was odd. Since these conditions were clearly not met in the film data, it was concluded the $h00, h=2n$ condition was due to a special arrangement of the molecules within the cell.

In order for the four molecules of phos-oxide to crystallize in a cell with space group symmetry $Pmcm$, which has eight general positions, the molecules must occupy the fourfold set of special positions on the mirrors. Since the molecule was assumed to have C_{3v} point group symmetry, this arrangement may be made. Of the two mirrors only one (to which a is normal) may be used when one considers the size of the molecule in connection with the dimensions of the two mirrors as defined by the lattice constants. Either of the a mirrors ($x=0$ and $x=1/2$) but not both (assuming no disorder) may be used. Since the greatest portion

(49 electrons) of the total scattering ability (77 electrons) lies on the mirror, and since the mirrors are in positions for which the geometrical part structure factor expression allows full contribution of the scattering factor, the intensities of the $h00$ reflections will be completely dominated by the scatterers on the mirrors. Their intensities, therefore, will fall off in intensity in the same manner that the scattering factor curves decrease with $\sin \theta/\lambda$. It may be seen, then, that if there were disorder in the crystal which would statistically use the $x=0$ mirror half the time and the $x=1/2$ mirror the other half, the scattering from one mirror would be 180° out of phase with the scattering from the other for $h00$ reflections, $h \neq 2n$, and the extra systematic extinctions would thus arise. It also appears that placement of all four molecules on the $6 \times 18 \text{ \AA}$ mirror is not a very energetically favorable arrangement. Therefore, assignment of $Pmcm$ would imply a disordered arrangement which should be considered only if no ordered structure could be found.

The fourfold set of general positions would have to be used if the space group P_2cm were assigned because the only mirror contained in this space group is of dimensions $6 \times 8 \text{ \AA}$, and it was felt that that was not enough area to incorporate two molecules. Therefore, assignment P_2cm is impossible since the structure factor expression for atoms

in general positions could not yield systematic h00 extinctions.

Assuming no disorder, the space group assignment was made to $Pmc2_1$. This non-centrosymmetric space group also has four general positions, but unlike $P2cm$ its mirrors (one at $x=0$, the other at $x=1/2$) are parallel to \underline{c} and are, therefore, of dimensions $7 \times 18 \text{ \AA}$ and are large enough to accommodate two molecules each. If the molecules were so arranged the situation would be that of the disorder case of space group $Pmcm$, and the systematic h00 extinctions could then be accounted for. Such an arrangement was assumed and was subsequently proven correct.

It is noted that the pair of symmetrically related molecules on the $x=0$ mirror are crystallographically independent of the pair on the $x=1/2$ mirror. This arrangement is desired on the part of the crystallographer (for small problems) because although more structural parameters need to be determined than in the case where there is only one molecule per asymmetric unit, there are twice as many chemically equivalent distances and angles which would be expected to lead to more accurate estimations of parameter errors.

A full octant of x-ray diffraction intensity data was gathered at room temperature with zirconium filtered molybdenum radiation from an approximately parallelepiped crystal of dimensions $0.13 \times 0.23 \times 0.25 \text{ mm}$. A General

Electric XRD-5 x-ray unit equipped with a single crystal orienter and a scintillation counter was used to measure the intensities of the reflections. A 100 second, $1.67^\circ - 2\theta$ moving-crystal-moving-counter scan technique (15) was employed. The same 2θ range for each reflection was scanned with an omega offset of 1.8° for background correction. The data taking began with a 3.0° take-off angle, but when peaks started overlapping, the take-off angle was reduced to 1° , and the data gathering was started again from the beginning. The reduction of the take-off angle gave the desired result of eliminating the peak overlap program. It is felt, however, that more serious problems were encountered because of the reduced angle when the crystal apparently shifted slightly. The collected data is thought to suffer slightly because of the lack of special precautions that should have accompanied the reduction of the angle.

Nearly all of the intensities of the 1600 reflections within a 2θ sphere of 65° were measured, but only those below 55° , 2θ were generally distinguishable from background and were used in the refinement.

Three standard peaks (0,6,0; 1,0,14; 0,0,16) were measured periodically throughout the data taking process for monitoring decomposition and crystal shifting. A general ten per cent decrease in intensity was observed. The intensities were corrected accordingly. Correction of the

intensities for lost counts due to non-linearity in the counting chain was made on the basis of 21 of the strongest reflections whose intensities were measured at one milliamp tube current as well as at the regular current (18 ma). The ratio of the intensities of the peaks at each setting were plotted against the measured peak at the data taking current. It was observed that the ratio became constant below a certain counting rate. All peaks above this counting rate were corrected according to the plot.

In addition, the intensities were corrected for the usual Lorentz-polarization

$$LP = \frac{1 + \cos^2 2\theta}{2 \sin 2\theta}$$

and also for non-characteristic ("streak") radiation. Since the non-characteristics radiation is a function of the mosaic character of the crystal under observation as well as the characteristics of the x-ray tube and the applied voltages, its correction is based on its experimentally measured distribution. The method and expressions derived by Benson and Fitzwater^x were used to correct the intensities of the reflections of order three or greater. A discussion of streak and its correction which is essentially the same

^xJ. E. Benson and D. R. Fitzwater, Ames, Iowa, Ames Laboratory, U.S. Atomic Energy Commission. Streak correction. Private Communication. 1964.

as that used has been reported by Williams and Rundle (16).

An estimate of the standard deviation of the intensity for each reflection was computed by means of the following expression:

$$\sigma_I^2 = TC + Bg + (P_T TC)^2 + (P_B Bg)^2 + (P_S St)^2.$$

TC, Bg and St refer to total counts, background and streak, respectively. The first two terms represent statistical uncertainty in the peak and background measurements. P_T , P_B and P_S refer to estimated relative errors in the measurement or calculations of peak, background and streak correction. For those reflections for which the corrected intensity was greater than zero, a standard deviation for the observed structure factor was computed by means of the expression

$$\sigma = \frac{\sigma_I Fo}{2I}$$

where I is the corrected intensity and Fo is the observed structure factor ($Fo = (I/LP)^{1/2}$). The rest of the unobserved reflections were given σ_F 's which were representative of the small but non-zero Fo's.

Structure Determination

The space group $Pmc2_1$ is polar with respect to z, thereby permitting an arbitrary origin in that direction. Assuming that the molecules lie on the mirrors at $x=0$ and $x=1/2$ by the reasoning discussed earlier, a phosphorus of one of

the molecules, at the outset, could be fixed with respect to x and z . A Patterson map was computed using all of the data by means of a Fourier summation program of Ledet (17). Unfortunately, the phos-oxide structure did not lend itself well to conventional Patterson techniques in that, first of all, there are numerous interatomic vectors which are parallel and which manifested themselves in terms of overlapping, smeared out peaks on the Patterson map. Secondly, because of the placement on the mirrors, not only are most of the interatomic vectors oriented in the y,z plane, but so also are most of the intermolecular vectors between the atoms of a molecule and its symmetrically related mate (also on the same mirror). The Harker line, $(0, y, 1/2)$, containing all of the $P-P'$, $O-O'$ and $C-C'$ vectors (the primed and the unprimed atoms are related by the \underline{c} glide in the \underline{b} direction), for example, was seen to contain a total of two large, smeared out peaks with a hint of a shoulder on one of the peaks. In an effort to make more use of the Patterson function the data was modified by means of a sharpening function and program of Granoff and Jacobson^{*}. This modification is in the form of multiplicative factors, applied to each structure factor value, which in

^{*}B. Granoff and R. A. Jacobson, Ames, Iowa, Ames Laboratory, U.S. Atomic Energy Commission. A data sharpening program. Private Communication. 1964.

essence straightens out the scattering factor curves with respect to $\sin \theta/\lambda$ as would be the case if the x-rays were scattered by point sources instead of finite sized distributions of scatterers. The result, in terms of the Patterson function, is sharper, less diffuse peaks. Still, however, there was a great deal of overlap.

The same overlap problem was felt to be the cause of the prevention of obtaining useful interpretations of attempted superposition techniques. The first step in the solution of the structure was finally made by proposing molecular orientations, which in conjunction with packing considerations, would give rise to a maximum number of vectors in a direction that would account for the very large peaks on the $(0, y, 1/2)$ Harker line and still be consistent with the other large peaks.

Once the orientations of the molecules were proposed, the two phosphorous locations were postulated and refined by least-squares techniques. The assumed molecular structure was used to propose the terminal oxygen sites and finally the other mirror-containing atoms. Further least-squares refinement and subsequent Fourier electron density syntheses (17) led to the inclusion of the remainder of the atoms.

Refinement of the structure was accomplished by means of a full matrix least-squares program of Fitzwater^x which minimized the quantity, $\sum w(|F_o| - |F_c|)^2$, where $w = \frac{1}{\sigma_F^2}$.

Hartree-Fock-Slater atomic scattering factor tables (18) were employed in the calculation of the structure factors. Anisotropic thermal parameters were eventually given and varied for each atom. It was felt that the quality of the data did not warrant the inclusion of hydrogen atoms.

Near the end of the refinement the data was ordered in terms of decreasing magnitude of F_o and a structure factor calculation was made in which an R-factor summary was computed every 50 reflections. A plot of the weighted R-factor vs. the average F_o of the set showed a linear decline in the R-factor with the decrease in F_o . It was assumed that with the proper weighting scheme the weighted R-factor would be constant for portions of data analysed in any significant systematic fashion (19). A linear correction which tended to remove the dependence was applied to the σ_F 's. It is felt that the standard deviations of the parameters were meaningfully improved with this adjustment.

The final value of the residual discrepancy index or R-factor

^xD. R. Fitzwater, Ames, Iowa, Ames Laboratory, U.S. Atomic Energy Commission. A crystallographic least-squares program for the IBM 7074. 1963.

$$R = \frac{\sum ||F_o| - |F_c||}{\sum |F_o|}$$

for relating 108 positional and thermal parameters to 781 observed reflections is 0.082.

The weighted R-factor

$$R_{wt} = \frac{\sum ||F_o/\sigma| - |F_c/\sigma||}{\sum |F_o/\sigma|}$$

is 0.078. The total 1191 reflections used in the refinement gave an unweighted R-factor of 0.0121 and a weighted R-factor of 0.105. A list of these calculated and observed structure factors is given in Figure 1. The final positional and thermal parameters and their standard errors from which the calculated structure factors were computed are listed in Table 1.

The Structure

A drawing of the molecular structure found for phos-oxide with accompanying pertinent interatomic distances and angles is given in Figure 2. The distances and angles and their standard errors were computed on the basis of the complete variance-covariance matrix of the last least squares cycle by means of a function and error program of Busing, Martin and Levy (20). The values of the distances and angles shown in Figure 2 are averages of the chemically equivalent but crystallographically independent structural features of the two molecules. A list of the individual interatomic

Figure 1. Comparison of calculated and observed structure factors for $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$ based on the structural parameters given in Table 1. The three columns consist of l , F_o (x10) and F_c (x10) on an absolute scale.

Dense numerical data table with columns including letters like L, FD, FC and various numbers. The data is organized in a grid-like structure with rows and columns of numbers and identifiers.

Table 1. Final positional and thermal parameters and their standard errors for $\text{OP}(\text{OCH}_3)_3\text{CCH}_3$. The anisotropic temperature factor is of the form $\beta_{11}h^2 + \beta_{22}k^2 + \beta_{33}l^2 + 2\beta_{12}hk + 2\beta_{13}hl + 2\beta_{23}kl$

Atom	x/a	y/b	z/c	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Molecule 1									
O ₁	0.0	0.1183 (.0017)	-0.0816 (.0005)	0.0608 (.0037)	0.0601 (.0043)	0.0029 (.0002)	0.0	0.0	0.0011 (.0008)
P	0.0	0.0598 (.0006)	0.0	0.0279 (.0007)	0.0383 (.0013)	0.0026 (.0001)	0.0	0.0	0.0010 (.0003)
O ₂	0.0	0.2632 (.0011)	0.0566 (.0011)	0.0336 (.0004)	0.0272 (.0022)	0.0035 (.0021)	0.0 (.0002)	0.0	0.0027 (.0005)
O ₃	0.1829 (.0009)	-0.0807 (.0011)	0.0253 (.0003)	0.0414 (.0020)	0.0528 (.0023)	0.0034 (.0002)	0.0097 (.0018)	0.0042 (.0004)	0.0010 (.0006)
C ₁	0.0	0.2006 (.0018)	0.1341 (.0006)	0.0400 (.0036)	0.0249 (.0031)	0.0030 (.0003)	0.0	0.0	0.0012 (.0008)
C ₂	.1858 (.0012)	-0.1473 (.0015)	0.1048 (.0004)	0.0340 (.0024)	0.0562 (.0036)	0.0032 (.0002)	0.0155 (.0023)	0.0020 (.0006)	0.0031 (.0007)
C ₃	0.0	-0.0538 (.0019)	0.1430 (.0005)	0.0212 (.0021)	0.0352 (.0034)	0.0021 (.0002)	0.0	0.0	-0.0013 (.0008)
C ₄	0.0	-0.1217 (.0021)	0.2280 (.0006)	0.0377 (.0034)	0.0527 (.0055)	0.0031 (.0003)	0.0	0.0	0.0050 (.0010)

Table 1. continued

Atom	x/a	y/b	z/c	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Molecule 2									
O	0.5	0.3946 (.0017)	0.1672 (.0004)	0.0451 (.0029)	0.0703 (.0046)	0.0023 (.0002)	0.0	0.0	-0.0003 (.0081)
P	0.5	0.4392 (.0006)	0.2466 (.0002)	0.0247 (.0007)	0.0396 (.0012)	0.0022 (.0001)	0.0	0.0	-0.0001 (.0003)
O ₂	0.5	0.6960 (.0013)	0.2681 (.0004)	0.0414 (.0028)	0.0336 (.0025)	0.0041 (.0003)	0.0	0.0	-0.0042 (.0007)
O ₃	0.3157 (.0010)	0.3443 (.0011)	0.2913 (.0003)	0.0369 (.0019)	0.0655 (.0029)	0.0043 (.0002)	-0.0232 (.0019)	0.0009 (.0005)	-0.0024 (.0006)
C ₁	0.5	0.7475 (.0022)	0.3500 (.0006)	0.430 (.0041)	0.0401 (.0044)	0.0033 (.0003)	0.0	0.0	0.0014 (.0010)
C ₂	0.3185 (.0011)	0.3868 (.0015)	0.3710 (.0004)	0.0302 (.0021)	0.0481 (.0033)	0.0036 (.0003)	-0.0172 (.0022)	0.0006 (.0006)	-0.0001 (.0007)
C ₃	0.5	0.5230 (.0017)	0.3902 (.0006)	0.0239 (.0024)	0.0286 (.0032)	0.0037 (.0003)	0.0	0.0	0.0003 (.0008)
C ₄	0.5	0.5726 (.0025)	0.4769 (.0005)	0.0336 (.0030)	0.0678 (.0059)	0.0022 (.0002)	0.0	0.0	-0.0051 (.0010)

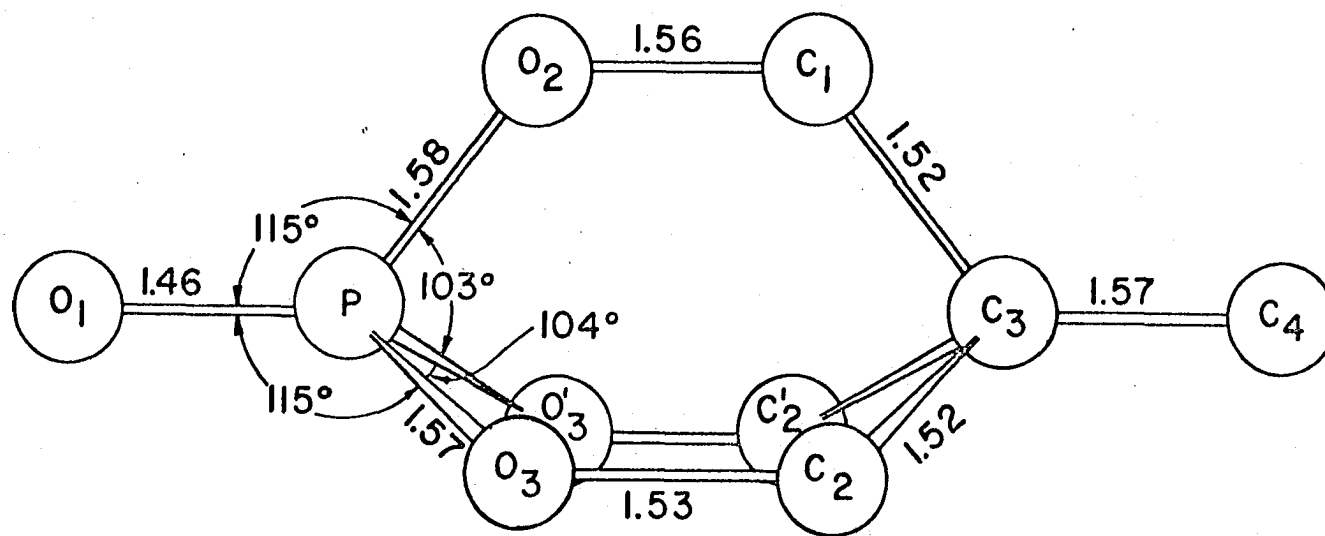


Figure 2. The molecular configuration of $OP(OCH_2)_3CCH_3$. The distances and angles are averages of the two crystallographically independent molecules of the asymmetric unit

distances and their standard errors as well as certain averages is given in Table 2. Table 3 contains a list of individual and averaged bond angles and their standard errors.

Large thermal parameters are common for most all of the atoms as might be expected from the soft crystal. The relatively large standard deviations as well as observed deviations in bond distances and angles reflect some of this thermal motion, but reflect, more so, the general quality of the data. The combination of decomposition, slight crystal shifting and inexperience (in perhaps the reverse order) in the data taking process led, it is felt, to the relatively large uncertainties in the finished product. More than one attempt was made to regrow fresh crystals of phos-oxide for the purpose of retaking the observed intensity data, but crystals of sufficient quality to warrant the project could not be found. The original crystal from which the intensity data was gathered was too badly decomposed to be used again.

The molecules of phos-oxide pack head to tail in a saw-tooth fashion on each mirror. A stereoscopic drawing of the molecules within the unit cell was computed in order to visualize the packing as well as the thermal motion of the atoms by means of OR TEP (21-22). The plotted result is given in Figure 3.

Table 2. Individual and averaged interatomic distances and their standard errors of $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$

Atoms	Molecule 1		Molecule 2		Average Dist. (A)	Average Std. Error
	Dist. (A)	Std. Error	Dist. (A)	Std. Error		
P-O ₁	1.491	0.017	1.436	0.015	1.464	0.016
P-O ₂	1.575	0.016	1.577	0.018	1.574	0.015
P-O ₃	1.556	0.012	1.578	0.012		
O ₂ -C ₁	1.425	0.022	1.488	0.024	1.455	0.020
O ₃ -C ₂	1.467	0.017	1.439	0.017		
C ₁ -C ₃	1.524	0.028	0.516	0.030	1.519	0.024
C ₂ -C ₃	1.528	0.018	1.506	0.018		
C ₃ -C ₄	1.565	0.023	1.569	0.024	1.567	0.023
P-C ₃	2.629	0.017	2.599	0.019	2.614	0.018

Table 3. Individual and averaged bond angles in degrees and their standard errors of $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$

Atoms	Molecule 1		Molecule 2		Average Angle	Av. Std. Error
	Angle	Std. Error	Angle	Std. Error		
$\text{O}_1\text{-P-O}_2$	116.2	1.0	114.6	0.9	115.0	0.8
$\text{O}_1\text{-P-O}_3$	114.0	0.5	115.3	0.7		
$\text{O}_2\text{-P-O}_3$	103.3	0.6	103.1	0.6	103.7	0.6
$\text{O}_3\text{-P-O}_3^{\text{a}}$	104.7		103.8			
$\text{P-O}_2\text{-C}_1$	114.5	1.2	115.9	1.2	115.2	1.1
$\text{P-O}_3\text{-C}_2$	115.7	0.9	114.9	0.9		
$\text{O}_2\text{-C}_1\text{-C}_3$	111.1	1.4	106.1	1.8	108.7	1.4
$\text{O}_3\text{-C}_2\text{-C}_3$	108.5	1.2	109.2	1.2		
$\text{C}_1\text{-C}_3\text{-C}_2$	108.5	1.2	111.6	1.2	109.6	1.2
$\text{C}_2\text{-C}_3\text{-C}_2^{\text{a}}$	109.9		108.5			
$\text{C}_1\text{-C}_3\text{-C}_4$	111.0	1.6	107.2	1.9	109.2	1.5
$\text{C}_2\text{-C}_3\text{-C}_4$	109.5		108.9	1.1		

^aPrimes refer to atoms related by the mirrors to the unprimed designation.

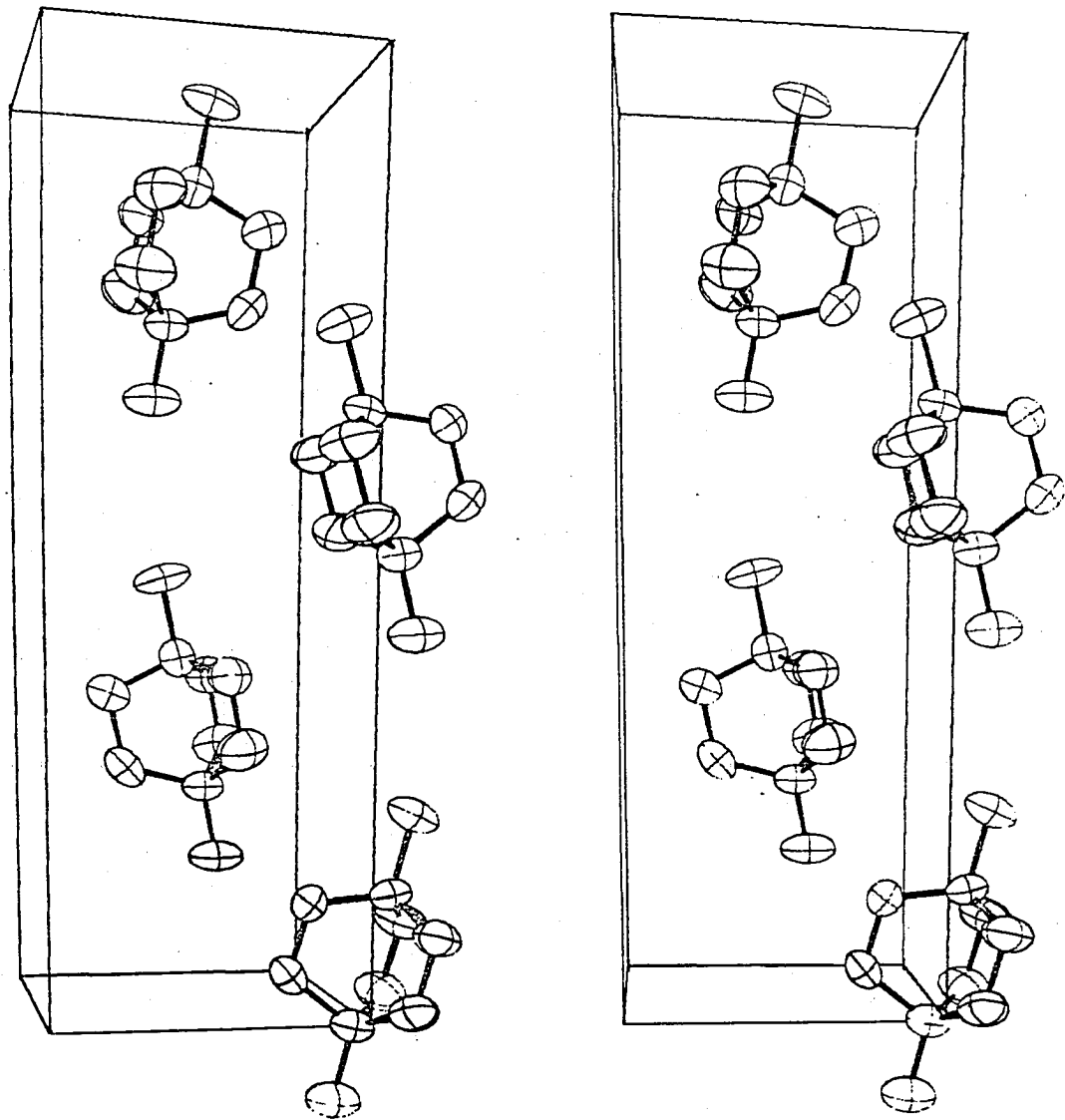


Figure 3. The atomic thermal ellipsoids (50 % probability) and the packing of the molecules of phosphorus oxide. The a axis runs horizontally to the left, and the c axis runs vertically upward

Discussion

The structure of phos-oxide clearly confirms the postulated molecular model. It thereby hopefully strengthens the use of IR and n.m.r. as structural tools in that the interpretation of their spectra was the basis of the molecular structure postulation.

The related bicyclic phosphate $OP(OCH_2)_3CC_2H_5$ has been reported by Neunhoeffer and Maiwald (23) to exist as dimers on the basis of their molecular weight determinations in benzene. Although this phenomena would not be entirely unexpected due to the large dipole moment, the molecules of phos-oxide in the crystal structure are clearly monomeric.

The dipole moment of phos-oxide has been observed to be 7.10 D (24). In the crystal the dipole moment vector of each molecule makes an angle of approximately 17° with the crystallographic c axis. Thus the component in the z direction is seen to be about 6.8 D. The electrical properties of crystalline electrets such as this have been discussed by Zhdanov (25). It would seem logical that, contrary to observation, such a high value of constant polarization would have an electric field observable in the formation of ice crystals on its surface.

The large dipole moment of the molecule probably accounts for the needle-like crystal habit. Whereas long, non-polar

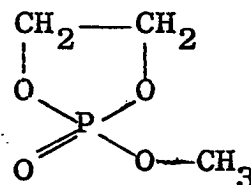
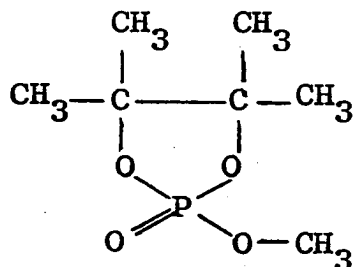
molecules usually pack parallel to one another so as to form plate-like crystals, the dipole moment forces of a molecule such as this exert more of an influence on the packing than does the van der Waals forces.

The P^{31} n.m.r. spectra of the unoxidized bicyclic phosphite $P(OCH_2)_3CCH_3$ has been examined in terms of quantitative structural properties (7) by means of an empirical formula proposed by Parks (26) for estimating the X-P-X angle in PX_3 compounds. The formula relates the P^{31} chemical shift, referred to phosphoric acid under standard conditions, to a calculatable quantity D which is a measure of the "number of unbalanced p electrons" in the phosphorus valence shell. Its value is a function of the electronegativities of P and X and of another quantity β whose square is directly related to the X-P-X bond angle and whose expression has been derived by Coulson (27, pp. 193 ff.) as a function of the degree of hybridization in the system. The expressions for these quantities are in the literature (26, 28) and will not be presented here. At any rate they allow one to estimate X-P-X angles on the basis of assigned electronegativities and measured P^{31} chemical shifts. Such a treatment yielded a value of 104° for the OPO angle in $P(OCH_2)CCH_3$.

Although a correlation between P^{31} chemical shifts and PO_4 type compounds like phos-oxide has not been presented,

the constraints of the bicyclic structure make it probable that the bond angle changes within the cage would not be affected greatly in the oxidation of the phosphite to the phosphate. For example, the OPO angles of P_4O_6 and of P_4O_{10} differ by only 2.5° (29, 30, 31). In spite of expected similarity of the OPO bonding between phos and phos-oxide the calculated OPO angle of 104° seems amazingly, and surely to some extent accidentally, close to the observed values of 103.7 found in phos-oxide. The chief reservation with Park's expression is that the calculated value depends significantly on the assigned electronegativities. Nevertheless, this structure determination does add another point to the curve for correlating structural properties with n.m.r. results. Certainly there exists a need for methods which give cheaper and more rapid measurements of structural features than x-ray analysis.

The crystal structures of two other trialkyl phosphates have been recently reported, methyl pinacol phosphate, $OP(OC(CH_3)_2)_2(OCH_3)$ (32) and methyl ethylene phosphate, $OP(OCH_2)_2(OCH_3)$ (33).



Since both of these compounds contain constrained five membered rings, the OPO and the POC angles are different from those found in phos-oxide. However, the 1.46 A P=O and the 1.57 A P-O bonds lengths found in phos-oxide agree within experimental error with the P=O and P-O distances reported in these two structures (1.44 and 1.57 A, respectively).

The OPO and POC angles found in phos-oxide (104 and 115°, respectively) as compared with those found in the above mentioned five membered ring compounds (99 and 112° respectively) indicate that less strain, as would be expected, is involved in this compound. On the other hand, the structure of an open chain dialkyl phosphate, dibenzyl phosphoric acid, $OP(OCH_2C_6H_5)_2(OH)$, has been reported by Dunitz and Rollett (34). Assuming the two ester groups in this structure represent a strainless system, it might be argued that the OPO and POC angles of this compound (104 and 120.5°, respectively) indicate that phos-oxide has some strain in the POC angles.

The parent caged phosphite, $P(OCH_2)_3CCH_3$, in a coordination compound in which four molecules of the phosphite are complexed with $AgClO_4$ has been studied by x-ray diffraction (35). The geometry is the same with the exception that the phosphorus bridgehead of the cage is more "pointed". The Ag-P-O angle is increased (from 115° (phos-oxide) to 118°), the OPO angle

is decreased (104° to 100°) and the POC angle has increased (115° to 122°).

The rare earth extractant properties of the related phosphate $\text{OP}(\text{OCH}_2)_3\text{CC}_5\text{H}_{11}$ (which is much less soluble in aqueous acid) have been studied by Goodman and Verkade (10). Tributyl phosphate has been previously shown to serve as a fairly good rare earth extractant. They found, however, that the extractant abilities of the caged phosphate were very poor and explained the phenomena on the basis of the strongly pi-bonded phosphoryl oxygen as evidenced by a very high P=O stretching frequency. They noted that the P=O stretching frequency of bicyclic phosphate was 1325 cm^{-1} as compared with 1260 cm^{-1} for the tributyl phosphate. Higher stretching frequencies are indicative of higher bond order which in turn might be thought of as withdrawal of electron density from the oxygen into the vacant d orbitals of the phosphorus, thereby explaining the decrease in extraction ability. The correlation between P=O stretching frequency and extractant ability has been noted also by Burger (36).

What is in question, however, is why the polycyclic phosphates have so much larger values for P=O stretching frequencies. It would be of interest to have accurate P=O distances at hand in order to construct a correlation table between stretching frequency and the P=O distance as has been done with carbonyl groups (37). Unfortunately, the only

three P=O distances known for trialkyl phosphate (all mentioned earlier) are within experimental error of one another.

In addition to the strong multiple bonding of terminal oxygen with the phosphorus some pi bonding between the ring oxygen and the phosphorus presumably exists as evidenced by the P-O bond lengths (1.57 A) which are significantly shorter than the sum of the covalent radii for phosphorus and oxygen (1.76 A) (38, p. 93). As the POC angle approaches 120° , indicative of sp^2 oxygen hybridization, the unhybridized lone pair p electrons become favorably positioned to donate to the vacant d orbitals on the phosphorus. The bond distances and angles involving phosphorus found in this compound have been correlated with several other sets in related phosphates and phosphites in Table 6 on page 67.

THE STRUCTURE OF ISOMER A

Introduction

Recently, Jenkins, Hutteman and Verkade reported the acid-catalyzed hydrolysis of the bicyclic phosphite 2,8,9-trioxa-1-phospha-adamantane, which yielded two isomeric products: isomer A (3- α -oxo-3- β -hydrido-7- β -hydroxyl-2,4-dioxa-3-phosphabicyclo[3.3.1]nonane) and isomer B (3- β -oxo-3- α -hydrido-7- β -hydroxy-2,4-dioxa-3-phosphabicyclo(3.3.1)-nonane) (39). The reaction is shown in Figure 4.

These two hydrolysates are among the few phosphorus acid diesters of the general type $OP(H)(OR)_2$ whose ligand properties with transition metals have been studied. Coordination of this type diester to the metal could take place through the oxygen (40), or it could take place directly through the phosphorus if the phosphoryl hydrogen were removed (41, 42, 43) or if the phosphoryl hydrogen migrated to the phosphoryl oxygen (44). In addition, coordination through the oxygen could take place even if the phosphoryl hydrogen was removed (45). Verkade and his co-workers reported that isomer A took the path whereby the phosphoryl hydrogen migrated to the phosphoryl oxygen in its coordination through the phosphorus with several transition metal salts. Evidence for this type of coordination was given by infrared studies which indicated

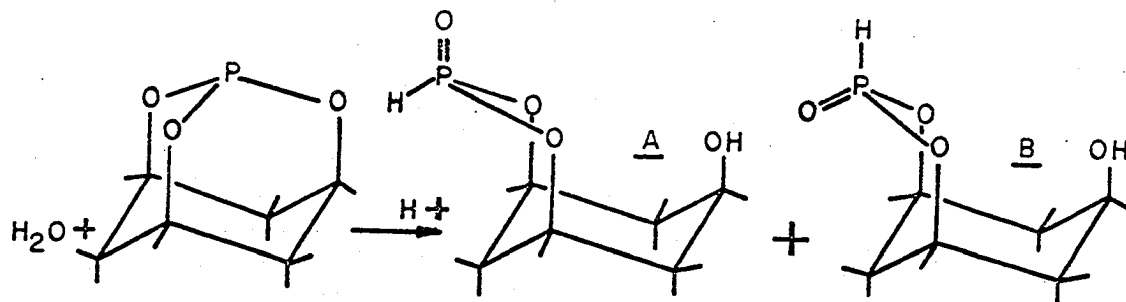


Figure 4. The acid-catalyzed hydrolysis of isomer A

a disappearance of the PH stretching mode upon complexation as well as the appearance of a new OH band.

The same study pointed out that in no case were metal complexes formed with isomer B. Modes and conditions for dehydration of the isomers back to the parent phosphite were also studied. Rationalization of these and other chemical and physical properties rely heavily on the postulated conformation and configuration of the two isomers.

Nuclear magnetic resonance and infrared absorption experiments have been used as tools for the postulation of the configuration and conformations of the two isomers. In their report (39) Verkade and his co-workers discussed the interpretation of these experiments. The proton n.m.r. spectra of isomer A is nearly identical to that of isomer B. The only major difference is that, whereas isomer A exhibits a PH doublet of single lines, isomer B exhibits a pair of 1331 quartets. It has been previously argued from geometrical factors that the presence of long range H^1-H^1 coupling could take place over five bonds (J_{HPOCCH} in this case) only if the bonds to the hydrogens were opposing and colinear (46). In this case the axial proton (down) of the methylene group directly below phosphorus could exhibit coupling with the phosphoryl hydrogen if the PH bond was colinear and opposed (up). The only conformation which would allow such a geometry is that shown for isomer B

in Figure 4. The more complicated splitting pattern of the PH hydrogen may then be explained by the additional coupling of the phosphoryl proton with each of the two methinyl protons and with the axial methylene proton directly below the phosphorus giving a pair of doublets of triplets which would appear as two 1331 quartets. Moreover, such a geometry might be expected to place the terminal oxygen in a position to hydrogen bond with equatorial hydrogen of the same methylene group. Hydrogen bonding has been known to influence the P=O stretching frequencies by 50-80 cm^{-1} when it has been to OH and NH groups (47). For isomer B it was found that, indeed, the P=O stretching frequency was lower than that for isomer A by an amount of 33 cm^{-1} (1241 and 1208 cm^{-1} respectively) lending support to the structure postulation. The P-H frequency also differs between isomers A and B implying an environmental change for the proton.

On the other hand the alcoholic OH stretching frequencies are the same (3450 cm^{-1}) for both isomers. The OH was thought to be axial on the basis of H-H coupling studies with n.m.r. An earlier, thorough study of an Arbuzov type ring cleavage of the same parent phosphite with RX conclusively showed from IR and n.m.r. arguments that X went onto the ring in the equatorial position (48, 49). The n.m.r. coupling of the axial proton of the carbon to which X was attached with the axial proton (trans to the first) of the adjacent carbon

was identified and measured. This same trans vicinal H-H coupling was found not to be present in the hydrolysates of the phosphite as would be expected if the OH occupied the equatorial position.

In summary, the lack of n.m.r. trans vicinal coupling - for either isomer indicates the axial site is occupied by the OH, and the constancy of the OH infrared stretching frequency for the two isomers supports the idea that the isomerism is not at the alcoholic group. That the isomerism takes place on the phosphoryl group is supported by the differences in stretching frequencies between the isomers for the P=O and the P-H bonds. That its form is as shown in Figure 4 is then evidenced by the long range H-H coupling for isomer B but not A and by the lowering of the P=O stretching frequency due to proposed hydrogen bonding in isomer B. Confirmation of the postulated conformation and configuration was one of the reasons for undertaking the x-ray structure of one of the isomers.

Additional interest in the crystal structure analysis stems from the fact there is little structural information concerning the stereochemical course of nucleophilic reactions of phosphites. Such knowledge would be expected to be particularly fruitful in the study of the stereochemistry of the reactions of polycyclic phosphites because, since the bond cleavage reactions do not fragment the molecule,

configurational and conformation analyses of the products of the reaction can be used to deduce the stereochemical aspects of the reaction (9). One problem in particular that might be raised is the question of the mechanism of the hydrolysis reaction of the parent phosphite as contrasted with the Arbuzov mechanism for its reaction with alkyl halides and halogens.

Experimental

Hydrolysis of 2,6,7-trioxa-1-phospha-adamantane is accomplished with the addition of a drop of perchloric acid to an acetone solution of equimolar quantities of the phosphite and H_2O . Separation of the two resulting isomers can be obtained by fractional crystallization from absolute ethanol (50). Dr. J. M. Jenkins kindly supplied crystals of isomer A using this procedure.

Single crystals suitable for x-ray analysis were grown from acetone by allowing the acetone to slowly evaporate from the solution. Several single crystals were selected from the batch and glued onto the ends of glass fibers with Duco cement. Both the film data and the intensity data were taken from crystals mounted in this manner. No protection from the atmosphere was found necessary. The crystal orientation was found in each case on an equi-inclination Weissenberg camera with $CuK\alpha$ radiation.

The Weissenberg camera with $\text{CuK}\alpha$ radiation was also used to gather $h k 0$ through $h k 4$ film data from one crystal and $0 k l$ through $3 k l$ data from another crystal. The observed diffraction symmetry from the films was $m m m$ (orthorhombic). Systematic extinctions were found for $h 0 0$, $h \neq 2n$; $0 k 0$, $k \neq 2n$; and $0 0 l$, $l \neq 2n$, which dictated uniquely the assignment of space group $P2_1 2_1 2_1$.

The lattice parameters were also obtained from Weissenberg photographs with $\text{CuK}\alpha$ radiation. Each photograph was individually calibrated by superimposing an aluminum powder diffraction pattern onto the film. After each photograph was taken and before the film was removed from its holder, the crystal was replaced with an aluminum powder sample. The x-ray port was then reopened and powder lines were recorded on the same film. The Weissenberg layer line screen was employed to limit the powder circles to 2 millimeter portions. The film holder was positioned in three places such that the film was calibrated in the middle region and on the two edges. $\text{K}\alpha_1$ spots were distinguishable from $\text{K}\alpha_2$ spots for both the aluminum powder pattern and for the single crystal of isomer A. Three different crystals were used to gather the data; two were mounted about the c axis and one about the a axis. The Bragg angles were carefully measured and calibrated. The lattice parameters and their standard deviations were obtained by a least-

squares treatment of the data (12) which employed a Nelson-Riley error function (51). The results are as follows:

$$a = 5.836 \pm 0.012 \text{ \AA}$$

$$b = 11.671 \pm 0.010 \text{ \AA}$$

$$c = 10.908 \pm 0.020 \text{ \AA}$$

It is not understood why the method did not yield more accurate lattice constants than it did.

An unsuccessful attempt was made to obtain accurate lattice parameters from a powder sample with a Guinier focusing camera with $\text{CuK}\alpha$ radiation. Westman and Magnéli (52-53) have reported that accurate lattice parameters can be obtained by a simple and rapid procedure with such a camera. They reported parameters for an orthorhombic structure accurate to ± 0.03 % . Whereas it is felt by this author that such accuracies for routine structure determinations are not that important, the need for a simple rapid procedure yielding reliable results does exist. Furthermore, since there is a Guinier focusing camera in the Ames Laboratory, it was decided to investigate this method.

Single crystals were finely ground and mixed with varying amounts of standard KCl. The lattice parameters of KCl are very well known, and the presence of it in the sample yields powder lines which can be used to calibrate the rest of the lines. Before the film was developed, an arbitrary

scale, for later measurement of the relative positions of the powder lines, was printed on the edge of the film.

Unfortunately, the powder lines of isomer A turned out to be rather diffuse; the $K\alpha_1$, α_2 doublet could not be resolved. It was felt that perhaps the soft crystals of isomer A became stressed during the grinding process. A new batch was therefore ground in a liquid nitrogen bath. No marked improvements resulted in the powder patterns, however, and the project was abandoned.

The calculated density for four molecules per unit cell is 1.59 gms/cm^3 . All atoms reside in general positions with one molecule per asymmetric unit. The structure determination therefore involves the determination of three positional parameters as well as the thermal parameters for eleven atoms excluding hydrogens.

A full octant of x-ray diffraction data was gathered at room temperature from a fresh crystal of dimensions $0.2 \times 0.2 \times 0.3 \text{ mm}$ in the shape of an approximate parallelepiped with zirconium filtered molybdenum radiation. A General Electric XRD-5 x-ray unit equipped with a single crystal orienter and a scintillation counter was used to measure the intensities.

Before the commencement of the data gathering, the settings of the adjustments of the counting chain electronics were checked, the size of the diffracted beam aperture was

determined, the take-off angle was selected, and the crystal was examined for twinning. A brief description of the criteria used for determining these parameters follows.

The counting chain consists of an adjustable high voltage d.c. power supply to the scintillation counter. The preamplified output of the photoelectric cell of the counter is sent to an amplifier. The amplifier has a pulse height discriminating capability by means of two adjustable controls. An adjustable threshold voltage cuts out pulses of energy less than its setting. An adjustable "window" voltage control cuts out pulses of energy greater than the sum of the threshold and the window voltages. The intermediate pulses are admitted to the scaler and ratemeter.

The 0, 0, 6 reflection, which gave a counting rate of approximately 1200 counts per second, was chosen for obtaining the data for the electronic settings. A plot of the counting rate as a function of the d.c. high voltage supply to the scintillation counter for a fixed threshold voltage with the window at a maximum value displays a plateau between the lower voltages where diffracted beam quanta are lost and the higher voltages where noise gets mixed in with the true signal. The proper high voltage setting is the center of the plateau. A profile of the $K\alpha$ peak (the distribution of energies of its pulses) is desired next so that the threshold may be set below the minimum energy of the $K\alpha$ pulses and the

window may be set to extend the opening through the peak to the maximum energy of the pulses. Such a profile may be obtained for the previously chosen high voltage setting by using a very small window and plotting the counting rate as a function of the threshold. The threshold and window may then be set to include any desired percentage of the $K\alpha$ peak. Since the width and position of the high voltage plateau is a function of the threshold, a recheck of the high voltage may be necessary if the chosen threshold is different from that used for the original high voltage selection.

A plot of the counting rate of a strong reflection as a function of the area of the aperture opening reveals, in essence, the size of the diffracted beam. The aperture size is chosen as small as possible but adequately large to allow the passage of the entire diffracted beam.

Pairs of reflections with similar χ and θ values may overlap in a normal θ scan if the take-off angle is too large. This is particularly troublesome with a fairly dense reciprocal lattice. One can, however, examine the calculated angles of the reflections for potentially troublesome reflections and scan them prior to taking data. Reduction of the take-off angle reduces the width of the peaks and the consequent overlap. The approximation of the scan measurement to the integrated intensity is in general better with the higher take-off angles, though.

By examining strip chart recordings of chi and omega scans of two or three reflections for splitting, one can detect slightly misoriented crystal fragments. It is noted, however, that precise twinning in which the reciprocal lattice points are more separated could pass this test. It is therefore advisable to take a zone of film data on the crystal selected for intensity data gathering.

All reflections within a 2θ sphere of 55° were scanned using a 100 second, $3.33^\circ - 2\theta$ moving-crystal-moving-counter scan technique (15) with a 3° take-off angle and a 1.8° aperture to the counter. In addition the same 2θ ranges for 160 reflections, including all within a 2θ sphere of 20° , were scanned with an omega offset of 1.8° for establishing tables to be used for background correction. These background intensity measurements were plotted as a function of 2θ . It was discovered that the background was a function of chi as well as 2θ ; therefore, two tables of the background as a function of 2θ were made up, one for chi between 0 and 25° and one for chi between 25 and 90° . These tables were used exclusively for the background of all reflections for which 2θ was greater than 16° . The actual observed background was used for reflections with 2θ less than 14° , and an average of the two was used between 14 and 16° .

The strip chart recording of each reflection was judged as to whether the intensity was clearly above background,

questionably above or indistinguishable from background. This information was coded for each reflection, and was carried through the data reduction to the cards containing the observed structure factors used for the structure determination and refinement.

Three standard peaks (0, 0, 6; 0, 6, 0; and 3, 0, 2) were periodically measured throughout the data taking process to monitor decomposition and crystal shifting. A linear least-squares fit of the decrease in intensity with x-ray exposure time was computed. (Higher order polynomial approximations were also computed, but no significant improvement was made on the least-squares fit.) Of the total 78 measurements taken only six fell outside of ± 1.0 % of the linear least-squares line indicating that the trends were statistically meaningful. The results were as follows:

0,0,6 (strongest reflection of the three):	5.7% decrease in intensity
0,6,0	: 11.5% decrease in intensity
3,0,2 (weakest reflection of the three)	: 1.5% increase in intensity
average	: 5.2% decrease in intensity

One would be hard pressed to predict the effect of decomposition on the rest of the reflections from these results. After the structure was determined, independent refinements of data with a 5 % decomposition correction and of data with no

decomposition correction were carried out. The uncorrected data gave better agreement than the corrected data.

Although it would be expected that most decomposition from x-ray exposure would be compound specific, it might be worthwhile expending some effort in an attempt to establish a criteria of some sort for selecting reflections to monitor decomposition. It was seen that the ones selected for this compound were of little value in measuring the general decomposition.

In addition to correction for background the intensities were corrected for non-linearity in the counter with the expression

$$I_{\text{corr}} = \frac{I_{\text{raw}}}{1 - I_{\text{raw}} \times 10^{-7}}$$

which corrects for the dead time of the counter electronics; for Lorentz-polarization ($LP = \frac{1 + \cos^2 2\theta}{2 \sin 2\theta}$); and for non-characteristic radiation ("streak") by means of the method of Benson and Fitzwater^x. No absorption correction was made due to the small linear absorption coefficient (3.36 cm^{-1}).

A standard deviation was calculated for the intensity of each reflection by means of the following expression:

$$\sigma_I^2 = TC + (0.03 TC)^2 + (0.03 Bg)^2 + (0.05 St)^2$$

^xJ. E. Benson and D. R. Fitzwater, Ames, Iowa, Ames Laboratory, U.S. Atomic Energy Commission. Streak correction. Private Communication. 1964.

where TC = total counts

Bg = background

St = streak correction

TC represents the square of the statistical counting uncertainty, and the last three terms of the expression represent estimates of non-statistical systematic errors in their values. The corresponding standard deviation for the observed structure was calculated with the following expression:

$$\sigma_F = \left(\frac{I + \sigma_I}{LP} \right)^{1/2} - \text{Fobs}$$

where I = corrected intensity

LP = Lorentz-polarization factor

Fobs = the observed structure factor = $\left(\frac{I}{LP} \right)^{1/2}$

The more straightforwardly derived expression,

$$\sigma_F = \frac{\text{Fobs} \cdot \sigma_I}{2I},$$

was not used because it becomes inapplicable when I approaches zero.

Solution of the Structure

A set of sharpened data was computed by means of a program of Granoff and Jacobson^{*}. Subsequently, both a

^{*}B. Granoff and R. A. Jacobson, Ames, Iowa, Ames Laboratory, U.S. Atomic Energy Commission. A data sharpening program. Private Communication. 1964.

sharpened and an unsharpened Patterson map were computed. The three Harker planes ($x=1/2$, $y=1/2$ and $z=1/2$) corresponding to the three screw axes were examined for large peaks representing phosphorus-phosphorus vectors. These Harker planes contain the P-P, O-O, and C-C vectors plus a few interatomic vectors that accidentally fall on these special planes. The ratio of the size of the P-P peaks to the O-O peaks should equal the ratio of the square of the number of electrons in P to the square of the number of electrons in O; one should, therefore, expect to be able to distinguish the P-P vectors from the rest.

A peak considerably larger than the rest, was indeed found on each of the three Harker planes. Furthermore, a single location in electron density space could be found for a phosphorus which could account for all three of the large peaks, and its position was thus established.

Spheres of 1.5 Å radius were contoured about each of the P-P peaks for locating the P-O vectors. In this manner an oxygen location which was consistent with all possible P-O vectors and O-O vectors on the Patterson was found. Several other potential oxygen positions were found, but none were completely consistent with all of the possible vector combinations. It is felt that peak overlap prevented exact establishment of their positions. The potential oxygen positions were rejected or established with alternating

least-squares position refinement and electron density computations. Subsequent Fourier synthesis in conjunction with the examination of a scale model of the postulated molecular structure suggested sites for the inclusion of the remainder of the atoms.

Refinement of the model was achieved by a least-squares minimization of the quantity $\sum w(|F_o| - |F_c|)^2$, where

$$w = \frac{1}{\sigma_F^2}, \text{ on an IBM 360/50 computer by means of ORFLS}$$

least-squares program (54) which employed Hartree-Fock-Slater scattering factor tables (18). With the inclusion of the last of the atoms the reliability index or R factor

$$R = \frac{\sum |F_o - s_q F_c|}{\sum |F_o|}$$

dropped from 0.191 to 0.084 in two cycles indicating the structure was indeed solved. It is of interest to note that this R factor value was computed with more than ten reflections per variable with isotropic temperature factors (two of which were not even allowed to vary at this point).

All atoms were eventually given anisotropic thermal parameters. Positions for the nine cyclohexane hydrogen atoms were calculated, and the atoms were included but were not allowed to vary. The phosphoryl and hydroxyl hydrogens were located on a difference Fourier synthesis. Their positions were adjusted to conform with typical bond distances and were not varied. When it was discovered that the

hydroxyl hydrogen might be weakly bonded to the terminal oxygen of a neighboring molecule, its position was allowed to vary. However, the shifts and standard deviations for its position were not meaningful, and the atom was returned to its calculated position.

Independent refinements with the 706 clearly observed reflections and with the total 1007 reflections were carried out, but the parameters and their standard deviations refined to very nearly the same values. The unweighted R factor for the final positional and thermal parameters (listed in Table 4) for the 706 observed reflections is 0.063. The weighted R factor

$$R_{wt} = \frac{(\sum w(F_o - s_q F_c)^2)^{1/2}}{(\sum F_o^2)^{1/2}}$$

is 0.052. Unweighted and weighted values for all reflections are 0.110 and 0.065, respectively. A listing of the observed and calculated structure factors based upon Table 4 is given in Figure 5.

It may be seen in Figure 5 that the calculated structure factors for the unobserved reflections are all very small and are frequently in contrast with the corresponding "observed" structure factors. Perhaps these 300 unobserved intensities should have been included with the 160 omega offset backgrounds in establishing the background tables. However, a structure factor calculation in which all

Table 4. Final positional and thermal parameters and standard errors (in parentheses) for isomer A. The anisotropic temperature factor expression is

$$\exp[-(h^2\beta_{11} + k^2\beta_{22} + l^2\beta_{33} + 2hk\beta_{12} + 2hl\beta_{13} + 2kl\beta_{23})]$$

Atom	x/a	y/b	z/c	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
P	0.24464 (0.00042)	0.14923 (0.00014)	0.11400 (0.00020)	0.02902 (0.00062)	0.00417 (0.00011)	0.00660 (0.00014)	0.00184 (0.00035)	-0.00044 (0.00038)	-0.00058 (0.00015)
O ₁	0.03964 (0.00100)	0.19142 (0.00042)	0.16795 (0.00050)	0.03904 (0.00223)	0.00906 (0.00049)	0.01147 (0.00060)	0.00512 (0.00090)	0.00083 (0.00097)	-0.00356 (0.00048)
O ₂	0.20607 (0.00080)	0.06706 (0.00036)	0.00445 (0.00040)	0.02985 (0.00188)	0.00519 (0.00032)	0.00685 (0.00043)	0.00328 (0.00073)	-0.00084 (0.00081)	-0.00004 (0.00034)
O ₃	0.40957 (0.00092)	0.08850 (0.00037)	0.20432 (0.00043)	0.03412 (0.00183)	0.00584 (0.00036)	0.00611 (0.00042)	0.00060 (0.00076)	-0.00183 (0.00087)	-0.00131 (0.00036)
O ₄	0.09120 (0.00100)	-0.12298 (0.00042)	0.18251 (0.00057)	0.02834 (0.00180)	0.00647 (0.00044)	0.01099 (0.00063)	0.00097 (0.00080)	0.00314 (0.00104)	0.00183 (0.00046)
C ₁	0.38308 (0.00136)	-0.01431 (0.00055)	-0.03544 (0.00057)	0.03147 (0.00272)	0.00606 (0.00057)	0.00447 (0.00055)	0.00254 (0.00114)	0.00132 (0.00111)	-0.00045 (0.00051)
C ₂	0.30228 (0.00115)	-0.13372 (0.00051)	-0.00470 (0.00059)	0.02769 (0.00277)	0.00487 (0.00050)	0.00577 (0.00056)	0.00085 (0.00097)	0.00053 (0.00100)	-0.00132 (0.00045)
C ₃	0.30024 (0.00120)	-0.16075 (0.00056)	0.13328 (0.00065)	0.02822 (0.00291)	0.00481 (0.00049)	0.00906 (0.00085)	0.00173 (0.00110)	0.00399 (0.00124)	0.00242 (0.00056)

Table 4. continued

Atom	x/a	y/b	z/c	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
C ₄	0.50614 (0.00119)	-0.11446 (0.00057)	0.19934 (0.00073)	0.02463 (0.00246)	0.00554 (0.00055)	0.00677 (0.00065)	0.00303 (0.00099)	-0.00056 (0.00120)	0.00113 (0.00056)
C ₅	0.58175 (0.00118)	0.00460 (0.00061)	0.16191 (0.00059)	0.01417 (0.00198)	0.00653 (0.00055)	0.00847 (0.00072)	0.00283 (0.00109)	-0.00322 (0.00109)	-0.00046 (0.00058)
C ₆	0.60597 (0.00134)	0.01370 (0.00054)	0.02532 (0.00066)	0.02305 (0.00244)	0.00525 (0.00057)	0.00896 (0.00075)	-0.00079 (0.00107)	0.00544 (0.00126)	0.00059 (0.00059)
H(P)	0.36607	0.23236	0.06841						
H(O ₄)	0.07135	-0.18462	0.22544						
H(C ₁)	0.40409	-0.01039	-0.12661						
H(C ₂)	0.14261	-0.14546	-0.03625						
H(C ₃)	0.31698	-0.24734	0.13566						
H(C ₄)	0.46979	-0.11241	0.28968						
H(C ₄)	0.63742	-0.16846	0.18607						
H(C ₅)	0.73546	0.02002	0.20071						
H(C ₆)	0.65353	0.09400	0.00307						

Figure 5. Comparison of calculated and observed structure factors for isomer A upon the parameters of Table 5. The three columns consist of k , F_o (x10) and F_c (x10). Negative values of F_o denote unobserved reflections

A large grid of numbers and alphanumeric codes, organized in columns and rows. The codes include letters like K, O, R, P, C, and numbers. The numbers range from small integers to larger values like 860, 133, and 860. The layout is dense and repetitive, typical of a data dump or a specific type of code book.

unobserved reflections were given the value of zero yielded unweighted and weighted R-factors of 0.145 and 0.078, respectively.

Structure

The molecular structure of isomer A is shown in Figure 6. A stereoscopic drawing of the molecule excluding the cyclohexane hydrogens was computed with a plotting program of Johnson (21 - 22) in order to visualize the thermal motion of the atoms. The plotted result is given in Figure 7. Interatomic distances and angles and their standard deviations were computed on the basis of the complete variance-covariance matrix of the last least-squares cycle with ORFFE (20). The results are listed in Table 5.

A stereoscopic drawing was also computed and plotted for the molecule and its symmetrically related neighbors within the unit cell to demonstrate the packing of the molecules. This drawing is shown in Figure 8. From this drawing and the table of interatomic distances (Table 5) one would presume that the packing makes use of a weak hydrogen bond between the hydroxyl hydrogen and O_1 of the neighboring molecule related by the twofold screw axis in the y direction. The O_4-O_1' distance (2.816 ± 0.007 A) is within the range of O-O systems which are designated as being hydrogen bonded (55). Moreover, the difference Fourier indicated that the hydroxyl hydrogen was between O_4 and O_1'

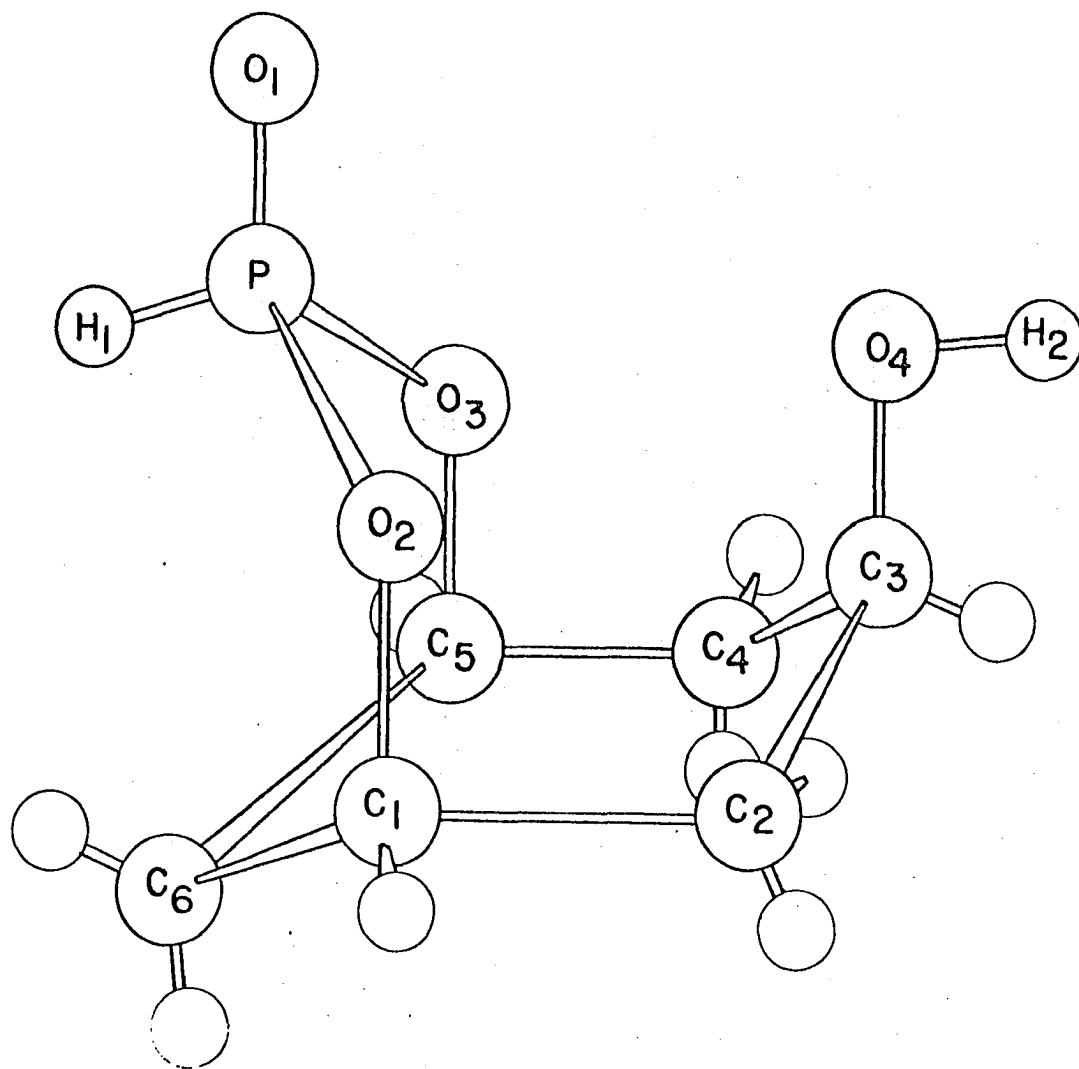


Figure 6. The molecular configuration and conformation of isomer A

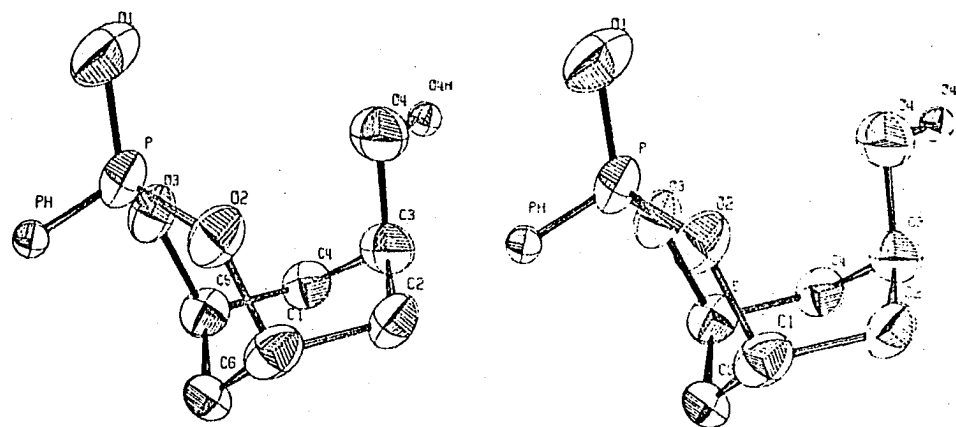


Figure 7. Stereoscopic view of the molecular structure and 50 % probability thermal ellipsoids of isomer A

Table 5. Interatomic distances and angles and their standard errors

Atoms	Bond Distances		Bond Angles		
	Distance	Standard Error	Atoms	Angle	Standard Error
O ₁ -P	1.420	0.006	O ₁ PO ₂	114.4	0.3
O ₁ -P ^a	1.446	0.006	O ₁ PO ₃	114.6	0.3
P-O ₂	1.550	0.005	O ₂ PO ₃	107.5	0.2
P-O ₃	1.549	0.005	PO ₂ C ₁	121.8	0.4
O ₂ -C ₁	1.468	0.008	PO ₃ C ₅	121.6	0.4
O ₃ -C ₅	1.476	0.008	O ₂ C ₁ C ₂	108.1	0.5
O ₄ -C ₃	1.403	0.009	O ₃ C ₅ C ₄	108.9	0.5
C ₁ -C ₂	1.507	0.009	O ₂ C ₁ C ₆	109.8	0.5
C ₂ -C ₃	1.541	0.009	O ₃ C ₅ C ₆	109.2	0.6
C ₃ -C ₄	1.501	0.010	O ₄ C ₃ C ₂	108.5	0.6
C ₄ -C ₅	1.512	0.009	O ₄ C ₃ C ₄	113.4	0.6
C ₅ -C ₆	1.503	0.009	C ₁ C ₂ C ₃	114.2	0.5
C ₆ -C ₁	1.495	0.010	C ₂ C ₃ C ₄	113.0	0.6
O ₄ -O ₁ ^b	2.816	0.007	C ₃ C ₄ C ₅	115.6	0.6
			C ₄ C ₅ C ₆	111.2	0.6
			C ₅ C ₆ C ₁	110.1	0.6
			C ₆ C ₁ C ₂	112.6	0.6

^aInteratomic distances averaged over thermal motion. O₁ is assumed to ride on P^{*} (56). The effects of thermal motion on all other interatomic distances was never more than 0.007Å.

^bO₁[†] is related to O₁ by the screw axis in y.

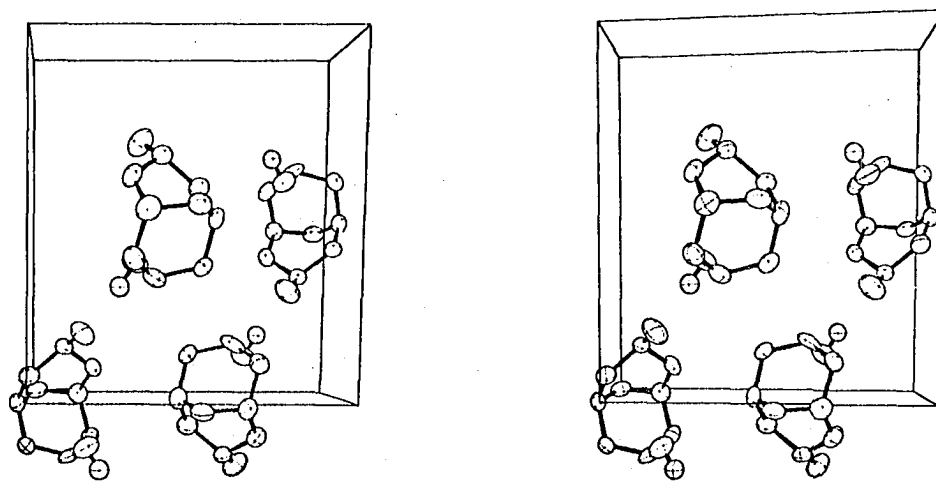


Figure 8. Stereoscopic drawing of the packing of isomer A. The c axis is horizontal and the b axis is vertical

and not between O_4 and O_2 and O_5 .

It may be seen qualitatively in the thermal-ellipsoid plots and quantitatively in the anisotropic temperature parameters (Table 4) and the calculated root mean square displacements that of all the atoms, O_1 has the largest thermal motion. The effect of thermal motion on the estimation of bond lengths from x-ray diffraction measurements has been discussed by Busing and Levy (56). They argue from the point of view that the most meaningful interatomic distance is the mean separation of the atoms. They have derived two expressions for thermal motion correction. One expression assumes that one atom of a bonded pair "rides" on the other and is applicable when the riding atom is much lighter than the other and is, in addition, strongly bonded to the other atom. The second expression assumes the atoms move independently. The first case seems most appropriate for the $P=O_1$ system. However, both corrections were computed and each gave the same correction (0.026 Å). Corrections for all of the other bond distances, assuming the atoms move independently, were computed, but no correction was greater than 0.007 Å.

The final value of the z coordinate of C_3 appears to be anomalous. The shift from its final value of several standard deviations in negative z direction tends to not only equalize the C_3-C_2 and the C_3-C_4 distances to the

average C-C distance in the structure but also to equalize the $O_4-C_3-C_2$ and $O_4-C_3-C_4$ angles. Various attempts were made to isolate a portion of the data responsible for its position or to find a chemically reasonable explanation of final position, but they were unsuccessful. For example, structure factor calculations were made on the refined parameters and also on the refined parameters except for the z coordinate of C_3 which was shifted negatively to a "more reasonable" value. The number of reflections for which δ

$$\delta = \frac{F_o - F_c}{\sigma}$$

improved by an amount greater than an arbitrary value, ϵ , with the shift in z were counted and were compared with the number which became worse by ϵ or more for the same shift. Furthermore, the magnitudes of F_o and F_c were compared. The results were that the δ 's improved by ϵ or more for 102 reflections of which 60 had $F_c > F_o$ and 42 had $F_o > F_c$, and that the δ 's got worse by ϵ or more for 118 reflections of which 65 had $F_o > F_c$ and 53 had $F_c > F_o$.

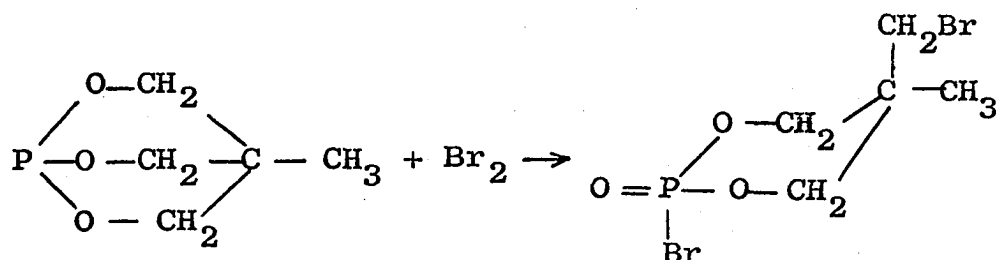
Discussion

The structure fully confirms the postulated conformation and configuration of isomer A. Furthermore, it strongly suggests that the proposed structure of isomer B is also correct. Of the two six membered rings in the structure the

phosphorus-containing one is in the boat form. It was originally thought that for isomer B this boat form could be stabilized by hydrogen bonding between the phosphoryl oxygen and the equatorial hydrogen of the methylene group below the phosphorus. Calculations based on the structure of isomer A seem to indicate otherwise. Assuming that the O=P-O angle and that the P=O distance in isomer B are the same as that found in isomer A (114.5° and 1.44 Å, respectively), the O...C distance is 3.44 Å. Additional calculations showed that decreasing the O=P-O angle to 110° shortened the O...C distance by only 0.2 Å. Although the energy requirements for such flexing are not known, it is doubtful that there could be enough energy gained by hydrogen bonding to pay for it, especially since an alkane-type CH group is a very poor participant in hydrogen bonding.

The boat form of the phosphorus containing ring is probably due first of all to the steric hinderances between the phosphoryl group and the alcoholic group which would prohibit a chair configuration for both the cyclohexane ring and the phosphorus-containing ring. It is probably more energetically favorable for the phosphorus ring to do the flipping than for the cyclohexane ring because hybridizational changes necessary at the intermediate stages are more easily made on the hinging oxygens than they are on four-

covalent carbon atoms. A comparison of the angles and amount of flipping might be analogous to the phosphite $\text{P}(\text{OCH}_2)_3\text{CCH}_3$ and its reaction product with Br_2 (57) in



which the flexing occurs at the oxygens. In comparing the parameters of the product with those of the phos-oxide structure it can be seen that less of an arc is involved in flipping the OPO plane. In addition, in the flipping of the carbon ring in isomer A unfavorable steric interactions occur in the eclipsing of several hydrogens. In cyclohexane, for example, the energy barrier for flipping from the chair form to the boat form is about 10 kilocalories of which only about 4.5 kcal are recovered in the boat form (58). Flipping of the OPO plane should involve almost no steric hinderance of the type involved at the carbons.

The structural parameters of the parent caged phosphite of isomer A have been measured by x-ray diffraction in the structure determination of the coordination compound $[\text{Ni}(\text{P}(\text{OCH})_3(\text{CH}_2)_3)_5](\text{ClO}_4)_2$ (59, 60, 61). One would assume that the changes in the parameters in going from the coordinated species to the free phosphite would be small, and that the parameters might, therefore, be used accordingly.

The OPO angle (107.5°) and the average POC angle (121.7°) found in isomer A are larger than those found by Riedel (103.5 and 116° , respectively). If his values are representative of the free phosphite it appears that in its hydrolysis to isomer A some strain in these angles is relieved; POC angles for uncyclized methoxy groups in phosphates, for example, have been recently reported with values near 120° (32, 33).

The multiple bond character of the P-O ring system as evidenced by the shortening of the P-O bond distance from the single bond value of the sum of the covalent radii has been discussed in the phos-oxide structure. It is interesting to note that isomer A has POC angles closer to 120° than does phos-oxide and correspondingly shorter P-O distances which supports the proposed bonding description discussed earlier.

The values of the phosphorus-oxygen distances and angles found for isomer A are compared with several other P-O systems in Table 6.

Clearly, the hydrolysis of the parent phosphite must occur by some mechanism other than that of its reaction with alkyl halides. The previously mentioned work concerning the Michaelis-Arbuzov reaction with alkyl halides (48, 49) gave well grounded evidence of the structure of the single isomer produced by the reaction. The conclusions of dipole moment and spectral experiments were that the halide occupied an

Table 6. Comparison of phosphorus-oxygen distances and angles of $\text{OP}(\text{OCH}_2)_3\text{CCH}_3$ with various organo-phosphorus compounds

Compound	$\text{OP}(\text{OCH}_2)_3\text{CCH}_3$	Isomer A	$\text{OP}(\text{OCH}_2)_2(\text{OCH}_3)$	$\text{OP}(\text{OC}(\text{CH}_3)_2)_2(\text{OCH}_3)$	P_4O_{10}
Reference			33	32	
P=O (A)	$1.46 \pm .02$	$1.44 \pm .01$	$1.44 \pm .01$	$1.44 \pm .01$	$1.40 \pm .02$
P-O (A)	$1.57 \pm .02$	$1.55 \pm .01$	$1.57 \pm .01$	$1.58 \pm .01$	$1.61 \pm .02$
O=P-O ($^\circ$)	$115.0 \pm .8$	$114.5 \pm .3$	$116.7 \pm .6^a$ $108.7 \pm .6^b$	$116.7 \pm .6^a$ $112.6 \pm .6^b$	116.5 ± 1.0
O-P-O ($^\circ$)	$103.7 \pm .6$	$107.5 \pm .2$	$99.1 \pm .6^c$ $107.5 \pm .6^d$	$98.4 \pm .5^c$ $105.6 \pm .7^d$	101.5 ± 1.0
P-O-C ($^\circ$)	115.2 ± 1.1	$121.7 \pm .4$	$112.0 \pm .9^e$ $118.8 \pm .9^f$	$112.2 \pm .8^e$ $123.4 \pm .9^f$	124.0 ± 1.0^g

^a To the ring oxygens.

^b To the methoxy oxygen.

^c Within the ring system.

^d Between the ring oxygens and the methoxy oxygen.

^e For the free methoxy system.

^f For the free methoxy system.

^g P-O-P angle.

Table 6. continued

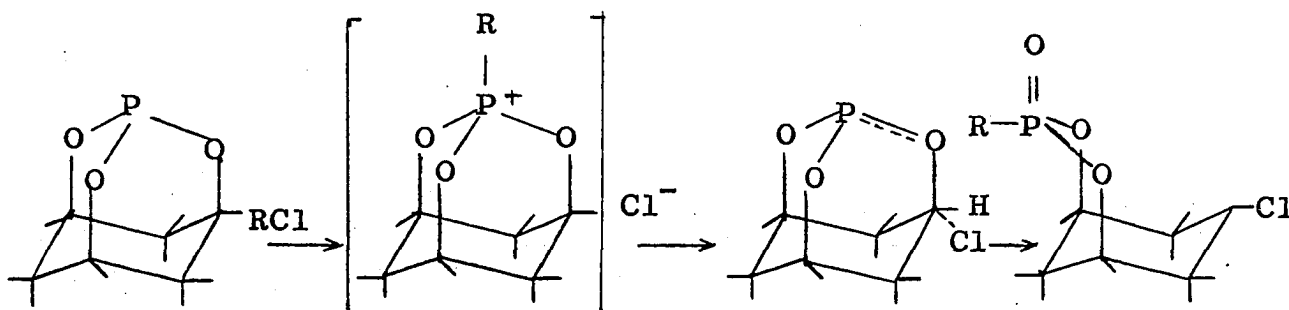
Compound	$OP(OCH_2Ph)_2(OH)$	BBMOD ^h	$Ag(P(OCH_2)_3CCH_3)_4ClO_4$	$Ni(PO_3C_6H_9)_5(ClO_4)_2$ ^j
Reference	34	57	35	61
P=O (A)	$1.47 \pm .005$	$1.46 \pm .02$	--	--
P-O (A)	$1.55 \pm .004$	$1.56 \pm .01$	$1.56 \pm .01$	$1.57 \pm .03$
O=P-O (°)	$110.6 \pm .2$	$113.8 \pm .9$	$117.9 \pm .6^i$	115.0 ± 1.0^i
O-P-O (°)	$103.8 \pm .2$	$105.5 \pm .7$	$99.9 \pm .9$	103.5 ± 1.5
P-O-C (°)	$120.6 \pm .3$	119.0 ± 1.0	122.0 ± 1.3	116.0 ± 1.5

^h 2-Bromo-5-bromomethyl-5-methyl-2-oxo-1,3,2-dioxaphosphorinane.

ⁱ M-P-O angle.

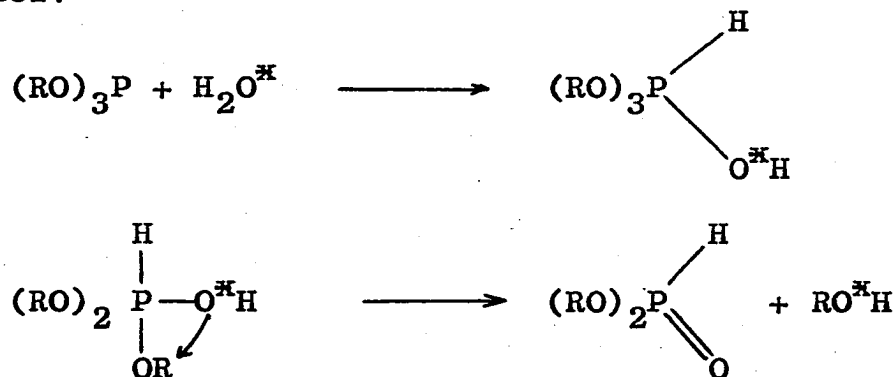
^j $[Ni(P(OCH)_3(CH_2)_3)_5](ClO_4)_2$

equatorial position on the ring in contrast with the axial alcoholic group in isomer A. In the Michaelis-Arbuzov reaction with RX an initial partial bond is made between



the phosphorus and the alkyl carbon forming in a quasi-phosphonium salt is expected as shown (62, 63). A $\text{S}_{\text{N}}2$ attack by the halide on the ring carbon could then cause the C-O bond to sever, and the phosphoryl group could then flip into the boat form.

This type of mechanism has been assumed by Arbuzov (64, 65) in the hydrolysis of trialkyl phosphite. An important feature of the proposed mechanism, as is shown, is that the oxygen of the water is carried away as the alcohol:



However, kinetic studies and isotopic labeling led Aksnes and Aksnes (66) to reject such a mechanism for the hydrolysis of tripropyl phosphite. Their experiments with ^{18}O -enriched water revealed that the labeled oxygen, as evidenced by displacements of the infrared P=O stretching bands, ended up as the phosphoryl oxygen. They suggested a plausible mechanism which was consistent with their kinetic data which indicated the reaction obeyed an overall third order rate equation, first order in the phosphite and second order in the water. Such a mechanism seems applicable for the hydrolysis of the bicyclic phosphite to isomers A and B. In contrast to the Arbuzov RX cage opening which can produce but a single isomer from this phosphite, the Aksnes type mechanism, shown in Figure 9, can allow for rearrangements at the phosphorus during an intermediate stage to account for the isomerism.

The same type kinetic studies and isotopic labeling experiments as carried out by Aksnes and Aksnes would be in order and, in fact, are in progress^{*} to test the proposed mechanism of this hydrolysis reaction.

^{*}J. G. Verkade, Ames, Iowa, Department of Chemistry, Iowa State University of Science and Technology. Current research. Private Communication. 1967.

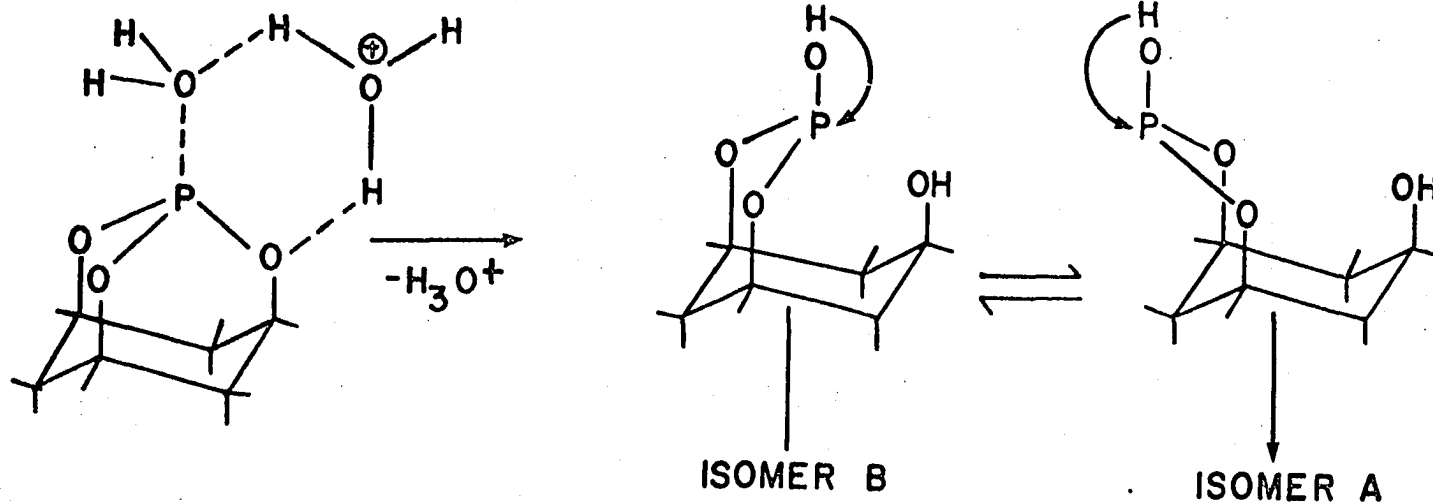


Figure 9. The proposed mechanism for the acid-catalyzed hydrolysis of 2,8,9-trioxa-1-phospha-adamantane

SUGGESTIONS FOR FUTURE WORK

A determination of the structure of one of the Arbuzov products of the reaction of 2,8,9-trioxa-1-phospha-adamantane with RX would provide information concerning the stereochemistry of this important class of reactions. With the careful IR and n.m.r. work that has been done on this type of compound in conjunction with the structures discussed in this report, it is pretty certain that the predicted conformation and configuration would be confirmed. Nevertheless, since a variety of RX may be used, the problem could be chosen to be a simple and straightforward heavy-atom problem with a suitable choice of X(CH₃I, for example, forms a crystalline product with the adamantane-like caged phosphate). Likewise, a lighter X could be chosen in order to obtain more accurate distances and angles not involving X.

It might be of interest to pursue the electrical properties of the phos-oxide compound more extensively. The pursuit would probably involve only a more thorough literature study of crystalline electrets than has been carried out by this author. It would seem, however, that such a large value of constant polarization as found in phos-oxide would be manifested in some interesting physical properties.

The handling of crystal decomposition in the x-ray beam for both phos-oxide and for isomer A was frustrating in that the particular reflections chosen to monitor the decomposition were apparently not representative of the radiation damage. It would seem logical that it might always be the case that any change in the internal structure would affect each reflection independently. No general guidelines could be found in the literature for handling the problem. It is conceivable that a study of decomposition in some particular cases might aid in proposing some guidelines that would not be altogether compound specific. Perhaps a more watchful eye on the backgrounds would provide an indication of decomposition. Perhaps the only conclusion one could make would be to suggest that film data be used to determine the order in which the counter data be taken.

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ACKNOWLEDGMENTS

The author is grateful to Dr. D. R. Fitzwater for his interest and guidance in this research and in the areas of computer usage and x-ray diffraction instrumentation. The author would also like to express his appreciation to Dr. J. G. Verkade for suggesting the research problems and for continuing his interest and support throughout the investigations.

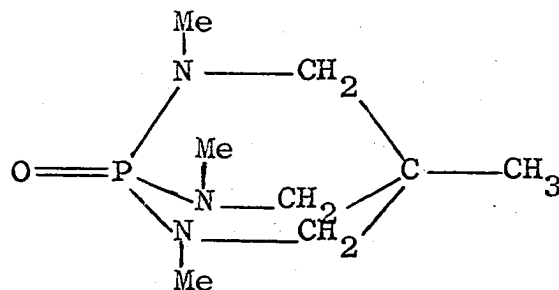
The assistance of Mr. Fred Hollenbeck in the x-ray laboratory is gratefully acknowledged. For the many helpful discussions concerning the theory and practice of x-ray diffraction the members of the x-ray chemistry groups over the past years are thanked.

Finally, the author would like to thank his wife, Sunni, for her continued support during graduate school and for her help in the preparation of this manuscript.

APPENDIX A. THE SPACE GROUP DETERMINATION AND
ITS IMPLICATIONS OF $OP(N(CH_3)CH_2)_3CCH_3$

Introduction

A structural analog of the phos-oxide compound which has aroused some chemical interest is 1-oxo-2,6,7-trimethyl-4-methyl-2,6,7-triaza-1-phosphabicyclo[2.2.2]octane. The proposed structural relationship to phos-oxide, made obvious in the following diagram, is that the bridging oxygens of the phos-oxide are replaced by N-CH₃ groups.

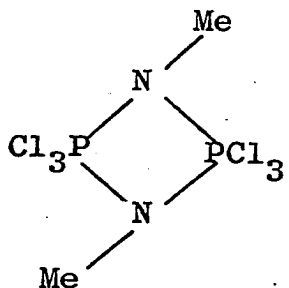


The molecule is apparently the same bicyclic, football shaped molecule with the probable retention of the threefold axis running through the molecule.

The chemical interest arises from the bidentate ligand properties of its parent phosphite and from the results of n.m.r. spectra, which together with the preparation of the compound, have been reported by Laube, et al. (67). They found the NCH₃ groups to be equivalent with respect to coupling of the P³¹ nucleus with their protons. The implication is that either the NR₃ configuration is locked

in a puckered arrangement, or the CH_3 group is rapidly interchanging with the lone pair of electrons on the N, or the NR_3 group is planar with a sp^2 type bonding orbital arrangement on the N.

Low temperature n.m.r. data indicated that there was no disorder locked into the molecule at lower temperatures which might be taken as evidence for the lack of inversion. On the other hand, the existence of planar NR_3 groups in cyclophosphonitriles is known. Recently, for example, Hoard and Jacobson reported the crystal structure of N-Methyltrichlorophosphinimine dimer (68) diagrammatically displayed as follows:



The two P's, the N and the methyl carbon are all in the same plane.

It was felt that if crystallographic evidence could be found supporting the planar configuration it would be a worthwhile contribution to the chemical literature concerning this class of compounds.

Structural Implications of Space Groups

It was noted that if the NR_3 part of this molecule was indeed planar there would exist a mirror plane coincident with the threefold rotational axis of the molecule. It would therefore be possible, although by no means necessary, for nature to make use of this mirror in the crystallization process for this compound. The close example of this is, of course, the phos-oxide molecule which crystallizes such that its molecular mirror is coincident with the mirror plane of the crystallographic space group.

Incorporation of molecular symmetry into the space group symmetry is a common occurrence. Furthermore, the detection of this phenomena can usually be accomplished experimentally by merely determining the space group, unit cell size, and number of molecules per unit cell. Taking these facts into account with the expected molecular symmetry— and the general size and shape of the molecule, one can make an attempt to conclude into which set of space group positions the molecule resides. For example, suppose that this particular compound crystallizes in a space group which contains a mirror and N general positions. Furthermore, suppose that the number of molecules per unit cell is $N/2$. One could then conclude that the molecule lies on the crystallographic mirror, must therefore contain a molecular mirror, and must therefore have a planar NR_3 group, since

there is no other apparent molecular symmetry other than the threefold axis.

Such an approach, it should be pointed out, is not without some pitfalls. First of all, the approach assumes that the general structure of the molecule is known whereas in fact that is what is to be determined. Secondly, this approach assumes that there is no disorder in the crystallization process. This has been seen, historically, to be a dangerous assumption. The longly disputed trimeric iron tetracarbonyl structure was thought to be linear with respect to the three irons because space group considerations implied a center of symmetry within the molecule. Dahl and Rundle (69) discussed this and gave crystallographic evidence of disorder in the form of shifted triangles of irons in a regular hexagon (with a center of symmetry). Later work has confirmed the triangular arrangement.

In terms of numbers, however, conclusions from this approach have usually turned out to be correct. In view of the chemical interest in the compound the elucidation of the space group was undertaken with this approach in mind. It was felt that even if no structural conclusions could be made from the space group the work might not be wasted in that the undertaking of a full crystallographic structure determination is being considered anyway. A determination of the space group at this point would shed light on the expected degree of difficulty of the crystal structure determination.

and would thereby have a bearing on a decision as to whether or not to support the full scale determination.

Experimental

Since the compound was reportedly hygroscopic, single crystals, provided by Mr. B. L. Laube from the synthesis mentioned earlier (67), were mounted in sealed Lindemann glass capillaries. The mounting was done in a dry nitrogen atmosphere inside a vacuum drybox. It should be mentioned, however, that later when one of the capillaries was accidentally broken, exposing the crystal to the atmosphere, only very slow decomposition with time was revealed in the subsequent x-ray photographs. This indicates that reasonable success would be expected if one mounted the crystals under room conditions and then evacuated and sealed the capillary when he was through.

The crystal was aligned on a Weissenberg camera, and zero through fourth layer Weissenberg photographs were taken. The diffraction symmetry was observed to be $m m m$ (orthorhombic). The conditions for non-extinction were as follows:

$$h k \ell \quad \text{none}$$

$$h k 0 \quad h = 2n$$

$$h 0 \ell \quad \ell = 2n$$

$$0 k \ell \quad k = 2n.$$

Accordingly, the assignment of the space group $Pbca$, which uniquely satisfies these conditions, was made.

Lattice constants, calculated from Weissenberg and rotation photographs, were as follows:

$$a = 13.15 \text{ \AA}$$

$$b = 13.49 \text{ \AA}$$

$$c = 12.99 \text{ \AA}$$

$$\text{volume} = 2306 \text{ \AA}^3$$

The lattice constant data was not calibrated and the results have an estimated uncertainty of 0.07 Å.

The density of the crystals was bracketed between 1.0 and 1.4 gms/cm³ on the basis of observing the floatation properties in l-bromo-propane and in H₂O. Such a density is consistent with the expectation that the density should be similar but slightly less than the phos-oxide compound (1.53 gms/cm³). The calculated density for eight molecules per unit cell is 1.17 gms/cm³.

Conclusions

Space group Pbc_a has eight general positions and two sets of special positions. The special positions are centers of symmetry. Since there is no molecular center of symmetry, one is forced to conclude that the molecules lie in general positions. Hence, it is impossible to make a conclusion concerning the planarity of the NR₃ group from space group considerations.

Remarks Concerning Full Scale Structure Determination

Excluding hydrogens there are 13 atomic positions to be determined. Certainly, as a crystal structure determination

problem, it could not be classified as complex. Furthermore, if the structure is solved, the three NR_3 group measurements are crystallographically independent and should yield more reliable conclusions with respect to the planarity as well as three independent, but chemically equivalent, P-N distances and related angles.

As with the phos-oxide compound, many of the interatomic vectors are parallel, and it may be difficult to solve the phase problem with conventional Patterson techniques of either analyzing the Harker lines or performing a general superposition. Because of the expected overlap it may be impossible to distinguish a P-P vector on a Harker line even though the problem borders on a heavy atom problem. (The general rule of thumb for such an approach is

$$\frac{(\text{atomic number of P})^2}{\Sigma(\text{at. no. of the rest})^2}$$

be greater than 0.5--the ratio is 0.44 for this compound.)

If one would be successful in determining a P position it is quite doubtful that it alone would dominate the signs of the structure factors enough to permit one to go directly to Fourier techniques. However, one superposition of the Patterson map using a minimum function should successfully solve the Patterson if the superposition point (u, v, w) is chosen as follows: $u = 2x$, $v = 2y$, $w = 2z$, where x, y and z are the fractional coordinates of the P in electron

density space.

If the phosphorus could not be located from the Patterson map, it is this author's suggestion that statistical direct methods for determining the signs of the structure factors be employed for solving the phase problem. This approach is suggested first of all because of the expected overlap and subsequent ambiguities of the Patterson map. Secondly, the fact that the compound crystallizes into a unique centrosymmetric space group with all atoms in general positions removes some of the problems of the direct method approach and gives a fairly high probability of success for the approach.

The total number of atoms per unit cell, and not the number per asymmetric unit, limits the degree of complexity of structure determinations by direct methods. It would be expected that for a crystal of this complexity with eight 13-atom molecules in the unit cell too many of the structure factors would be small, rendering the use of strict Harker-Kasper inequalities (70) ineffective in determining the structure. However, the basic sign relationship of Sayre (71), $s(\vec{h})s(\vec{h}')s(\vec{h}+\vec{h}') = +1$, where \vec{h} and \vec{h}' are the Miller indices of any two reflections, which is true for large structure factors and can be derived from Harker-Kasper inequalities, is "probably" true for smaller structure factors. A quantitative expression for this probability has

been sought by several workers. Cochran and Woolfson (72) have derived the following expression for the probability of Sayre's equation holding true:

$$P_+ (\vec{h}, \vec{h}') = 1/2 + 1/2 \tanh \frac{\epsilon_3}{\epsilon^3} |U_{\vec{h}} U_{\vec{h}'}, U_{\vec{h}+\vec{h}'}|$$

where U is the unitary structure factor

$$\epsilon = \sum_{j=1}^N n_j^2 \quad (n_j \text{ is the unitary scattering factor})$$

$$\epsilon_3 = \sum_{j=1}^N n_j^3$$

The use of this expression and others have been programmed for the computer. The National Research Council of Canada, for example, has a well documented program (73) which is available for general use. It is felt that only a moderate amount of work would be required to set up and hopefully solve this structure with such an approach.

APPENDIX B. AN ALGORITHM FOR THE TUNING OF A GENERAL
DIFFRACTION MAXIMA ON AN AUTOMATED FOUR-CIRCLE
DIFFRACTOMETER

A diffracted beam of x-rays or neutrons of a particular reflection is said to be tuned when the crystal and detector are positioned such that not only is the diffracted beam of maximum intensity but also that the maximum strike the center of the detecting system. The tuning of a general reflection on a typical single crystal orienter (15, 74) by some iterative process of adjusting the three crystal orienting rotational axes, chi, phi, and omega, and the detector axis, theta, is a process that is surprisingly difficult to define. One can easily get into an iterative process that can be carried on indefinitely without ever converging on the solution.

With a computer intimately available during the tuning process it has been possible to define an algorithm which will converge for any general reflection. Furthermore, the operations required in terms of shaft movements are very simple if one can make some arithmetic calculations during the process.

It was noted that when a crystal is in a position to diffract the "reflecting" plane (to which the diffraction vector is normal) is vertical. It was further noted that

omega axis^x is always vertical and is hence parallel with the plane, and that the chi axis is perpendicular to the omega axis and is nearly parallel with the plane if omega is nearly equal to the Bragg angle, theta. The upshot is that any plane can be properly positioned with chi and omega adjustments to give a maximum intensity in the horizontal plane within which the detector swings. This means that a general peak can be tuned by a iterative process of the following three steps:

1. Maximize the total intensity with respect to omega adjustments.
2. Center the diffracted beam with respect to top-bottom beam splitters on the detector face with adjustments in chi.
3. Center the detector on the diffracted beam with respect to left-right beam splitters on the detector face with adjustments in theta.

Phi is left out of the tuning process because it in general is not parallel or perpendicular to the reflecting plane, and only two axes are necessary to orient the reciprocal lattice vector. In particular, phi is parallel to the reflecting plane at $\chi = 0^\circ$ and is normal to the

^xThe axis lables, omega, chi, phi and theta are used in the conventional definition for single orientor units. See references 15 or 74, for example.

reflecting plane at $\chi = 90^\circ$ providing $\omega = \theta$, but it is in between for the rest of the cases.

The χ - ω tuning process will in general leave ω at some value other than the value of θ . The computer can then be used to calculate the χ and ϕ adjustments necessary to compensate for a rotation of ω to the Bragg angle. The derivation of the expressions for the compensating amounts will now be presented.

The following reference coordinate system will be used throughout the derivation. Consider a crystal in position to diffract.

\vec{r}_1 is coincident with the diffraction vector, normal to the "reflecting" plane, bisecting the incident and diffracted beams.

\vec{r}_3 is coincident with the vertical instrument axis ω .
 \vec{r}_2 is mutually perpendicular to \vec{r}_1 and \vec{r}_3 and is in the direction to make a right handed set.

Note that if $\omega = \theta$ \vec{r}_2 coincides with the χ axis. Furthermore, at $\chi = 0$ \vec{r}_3 and the ϕ axis are coincident. Moreover, the senses of \vec{r}_2 and \vec{r}_3 are the same as those under which χ and ϕ physically move.

A reciprocal lattice vector \vec{x} may be brought into coincidence with the diffraction vector \vec{d} of a given reflection (i.e. the crystal is brought into position to diffract) with appropriate ϕ , χ and ω rotations of

ϕ , χ and $-\Delta\omega$ from $\phi=0$, $\chi=0$ and $\omega =$ the Bragg angle.

It is again noted that in the initial conditions the ϕ and χ axes are coincident with \vec{r}_3 and \vec{r}_2 respectively and that the χ - \vec{r}_2 coincidence is independent of ϕ rotation. Hence, rotation of ϕ , χ and ω in that order may be done straightforwardly in the basic coordinate system.

$$\vec{d} = R(-\Delta\omega)R(\chi)R(\phi)\vec{x}$$

where the R's are the rotation operator matrices in the basic coordinate system.

$$R(-\Delta\omega) = \begin{pmatrix} \cos-\Delta\omega & \sin-\Delta\omega & 0 \\ -\sin-\Delta\omega & \cos-\Delta\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \Delta\omega & -\sin \Delta\omega & 0 \\ \sin \Delta\omega & \cos \Delta\omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(\chi) = \begin{pmatrix} \cos \chi & 0 & \sin \chi \\ 0 & 1 & 0 \\ -\sin \chi & 0 & \cos \chi \end{pmatrix}$$

$$R(\phi) = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It should be noted at this point for later manipulations that for this type of rotation matrices

$$R(-\alpha) = R(\alpha)^{-1} = R(\alpha)^T.$$

The $\omega = \theta$ solution of the problem occurs at $(0, \chi_0, \phi_0)$. Hence, in addition

$$\vec{d} = R(\chi_0)R(\phi_0)\vec{x}.$$

$$\begin{aligned}\vec{x} &= (R(-\Delta\omega)R(\chi)R(\phi))^{-1} \vec{d} = (R(\chi_0)R(\phi_0))^{-1} \vec{d} \\ &= R(-\phi)R(-\chi)R(\Delta\omega)\vec{d} = R(-\phi_0)R(-\chi_0)\vec{d}.\end{aligned}$$

Since \vec{d} is coincident with \vec{r} and may be regarded as a unit vector it may be represented by $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

$$\begin{aligned}\therefore \vec{x} &= \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \chi & 0 & -\sin \chi \\ 0 & 1 & 0 \\ \sin \chi & 0 & \cos \chi \end{pmatrix} \begin{pmatrix} \cos \Delta\omega & \sin \Delta\omega & 0 \\ -\sin \Delta\omega & \cos \Delta\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos \phi_0 & -\sin \phi_0 & 0 \\ \sin \phi_0 & \cos \phi_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \chi_0 & 0 & -\sin \chi_0 \\ 0 & 1 & 0 \\ \sin \chi_0 & 0 & \cos \chi_0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\end{aligned}$$

yielding

$$x_1 = \cos \phi \cos \chi \cos \Delta\omega + \sin \phi \sin \Delta\omega = \cos \phi_0 \cos \chi_0 \quad (\text{I})$$

$$x_2 = \sin \phi \cos \chi \cos \Delta\omega - \cos \phi \sin \Delta\omega = \sin \phi_0 \cos \chi_0 \quad (\text{II})$$

$$x_3 = \sin \chi \cos \Delta\omega = \sin \chi_0 \quad (\text{III})$$

These three equations contain the solution for the compensating chi and phi adjustments for a theta-omega difference of $\Delta\omega$. The chi correction is given directly by III:

$$\sin \chi_0 = \sin (\chi + \Delta\chi) = \sin \chi \cos \Delta\omega \quad (\text{IV})$$

where χ is the tuned chi value and χ_0 is the corresponding chi value at $\omega = \theta$. $\Delta\chi$, therefore, is the chi correction.

Dividing II by I gives an expression for $\tan \phi_o$:

$$\tan \phi_o = \frac{\sin \phi \cos \chi \cos \Delta\omega - \cos \phi \sin \Delta\omega}{\cos \phi \cos \chi \cos \Delta\omega + \sin \phi \sin \Delta\omega}.$$

Dividing numerator and denominator by $\cos \phi \cos \chi \cos \Delta\omega$ have

$$\tan \phi_o = \frac{\tan \phi - \frac{\tan \Delta\omega}{\cos \chi}}{1 + \frac{\tan \Delta\omega \tan \phi}{\cos \chi}}. \quad (V)$$

$$\text{But also } \tan \phi_o = \tan (\phi + \Delta\phi) = \frac{\tan \phi + \tan \Delta\phi}{1 - \tan \Delta\phi \tan \phi}. \quad (VI)$$

Equating V and VI have

$$\tan \Delta\phi = - \frac{\tan \Delta\omega}{\cos \chi}. \quad (VII)$$

Although VII is a compact, simple expression to evaluate, its derivation required division by $\cos \chi$, and is thereby rendered invalid at $\chi = 90^\circ$. An alternative expression was developed without any such invalid intermediate steps and is presented here, also.

I times II gives

$$\begin{aligned} & \sin \phi \cos \phi \cos^2 \chi \cos \Delta\omega + \sin^2 \phi \cos \chi \cos \Delta\omega - \cos^2 \phi \cos \chi \\ & - \cos^2 \phi \cos \chi \cos \Delta\omega \sin \Delta\omega - \sin \phi \cos \phi \sin^2 \Delta\omega = \\ & \cos \phi_o \sin \phi_o \cos^2 \chi_o. \end{aligned}$$

Replacing $\sin \phi \cos \phi$ with $\frac{\sin 2\phi}{2}$, collecting terms and replacing $\cos^2 \phi - \sin^2 \phi$ with $\cos 2\phi$, and finally dividing through by $\cos^2 \Delta\omega$ have

$$\sin 2\phi (\cos^2 \chi - \tan^2 \Delta\omega) - 2 \cos \chi \tan \Delta\omega \cos 2\phi = \frac{\sin 2\phi_o \cos \chi_o}{\cos^2 \Delta\omega}.$$

But from III

$$\cos^2 \chi_0 = 1 - \sin^2 \chi \cos^2 \Delta\omega, \text{ giving}$$

$$\frac{\cos^2 \chi_0}{\cos^2 \Delta\omega} = \tan^2 \Delta\omega + \cos^2 \chi.$$

$$\therefore \sin 2\phi_0 = \frac{\sin^2 \phi (\cos^2 \chi - \tan^2 \Delta\omega) - 2 \cos \chi \tan \Delta\omega \cos 2\phi}{\cos^2 \chi + \tan^2 \Delta\omega}.$$

Letting $k = \frac{\cos \chi}{\tan \Delta\omega}$ have

$$\sin 2\phi_0 = \frac{k^2 - 1}{k^2 + 1} \sin 2\phi - \frac{2k}{k^2 + 1} \cos 2\phi. \quad (\text{VIII})$$

But also

$$\sin 2\phi_0 = \sin(2\phi + 2\Delta\phi) = \sin 2\phi \cos 2\Delta\phi + \cos 2\phi \sin 2\Delta\phi. \quad (\text{IX})$$

Equating the coefficients of $\sin 2\phi$ and of $\cos 2\phi$ gives a valid solution to equations VIII and IX simultaneously for all values of k , yielding

$$\cos 2\Delta\phi = \frac{k^2 - 1}{k^2 + 1} \quad (\text{X})$$

$$\sin 2\Delta\phi = \frac{-2k}{k^2 + 1} \quad (\text{XI})$$

It may be seen that the necessary condition $\cos^2 2\Delta\phi + \sin^2 2\Delta\phi = 1$ is upheld.

Of the two solutions satisfying XI at $\chi = 90^\circ$, $\Delta\phi = 0^\circ$ and $\Delta\phi = 90^\circ$, the latter one is applicable. It may be seen that the $\Delta\phi$ correction ranges from $-\Delta\omega$ at $\chi = 0^\circ$ to 90° at $\chi = 90^\circ$. The range of $\Delta\chi$ is 0° at $\chi = 0^\circ$ to $\Delta\omega$ at $\chi = 90^\circ$. The expressions IV and XI are valid for $\pm\Delta\omega$, $\pm\chi$ in the

range $0^\circ < |\chi| < 90^\circ$. For $|\chi| > 90^\circ$ the $\Delta\phi$ expression gives the correct result, but the negative supplement of χ ($\chi - 180^\circ$) must be used in the $\Delta\chi$ computation. The general nature of the functions are summarized in Table 7.

Table 7. The nature of the general tuning expressions

χ	$\Delta\omega$	$\Delta\phi$	$\Delta\chi$
$>90^\circ$	+	+	+
	-	-	-
$+90^\circ$	$+\alpha$	+ or $- 90^\circ$	+ or $-\alpha$
	$-\alpha$	+ or $- 90^\circ$	- or $+\alpha$
$0^\circ < \chi < 90^\circ$	+	-	-
	-	+	-
0°	$+\alpha$	$-\alpha$	0°
	$-\alpha$	$+\alpha$	0°
$-90^\circ < \chi < 0^\circ$	+	-	+
	-	+	+
-90°	$+\alpha$	+ or $- 90^\circ$	- or $+\alpha$
	$-\alpha$	+ or $- 90^\circ$	+ or $-\beta$
$<90^\circ$	+	+	-
	-	-	-

The general tuning algorithm is therefore followed by rotating ω to equal θ and then calculating the required compensating $\Delta\chi$ and $\Delta\phi$ using expressions IV and XI, respectively.

Implementation of this algorithm has been made on a Hilger and Watts four-circle x-ray diffractometer. A comparison of the experimental findings with the calculated corrections for a particular reflection is presented in Table 8. Not only is the appropriateness of the expression verified, but also credit is made to the sensitivity of the instrument.

Table 8. Comparison of calculated and observed angle settings for a tuned reflection at $\theta = 8.462^\circ$

Run	Omega tuned ^a	Chi tuned ^a	Phi	$\Delta\phi$ calc. ^b	$\Delta\phi$ obs. ^c	$\Delta\chi$ calc. ^d	$\Delta\chi$ obs. ^e
1	8.601	62.190	74.700	0.298	--	0.0	--
2	8.535	62.039	254.700	0.156	--	0.0	--
3	7.665	62.212	76.700	-1.709	-1.702	-0.011	-0.022
4	7.593	62.048	256.700	-1.853	-1.844	-0.013	-0.009
5	3.722	62.590	85.200	-10.210	-10.202	-0.376	-0.400
6	3.645	62.431	265.200	-10.341	-10.344	-0.387	-0.392

^aThe tuned omega and chi values are averages of three experimental tunings of the reflections for runs 1, 2, 5 and 6 and are averages of four tunings for runs 3 and 4. The estimated standard deviations are $\pm 0.010^\circ$ for omega and $\pm 0.03^\circ$ for chi.

^bCalculation made by means of expression IV.

^c $\Delta\phi = \phi_1 \text{ or } 2 \text{ (calc.)} - \phi \text{ (obs.)}$.

^dCalculation made by means of expression XI.

^e $\Delta\chi = \chi_1 \text{ or } 2 \text{ (calc.)} - \chi \text{ (obs.)}$.